so also $F \cdot s = \Delta E$ & $F = \frac{\Delta E}{\Delta s}$

$$K_1 + U_1 + W_{conservative} = K_2 + U_2$$

$$U_{spring} = \frac{1}{2} k x^2$$

$W = F \cdot s = \Delta E$ (can be U or K)

$$KE = \frac{1}{2} m v^2$$

$$P = \frac{\Delta W}{\Delta t} = \vec{F} \cdot \vec{v}$$

$$U_{gravity} = mgh$$

Conservation of Energy (ch. 6 & 7)

If only conservative forces (i.e. gravity or springs) are acting in a system all E is conserved and constant. So $\Sigma E = KE + U$ and $E_0 = E_f$ initial E is same as final E

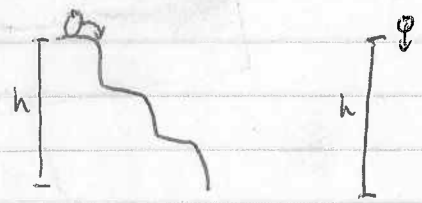
see bottom of p.261 & p.262 → Energy that is not conserved is lost through forces that take E out of the system as heat, sound, or some other unrecoverable quantity.

* Friction, drag both are forces that do negative work in both directions and are not conservative F.

Work is the amount of E a F puts into the system. It can be recovered (the E) as long as the force is conservative.

Energy is a scalar and so path does not matter.

In equations:



both balls start with $U = mgh$

and both end with same $\vec{V}$ where $\vec{V} = \sqrt{2gh}$ here's why

$K_1 = 0$

$U_f = 0$

$$E_0 = E_f$$
$$K_1 + U_1 = K_2 + U_2$$
$$0 + mgh = \frac{1}{2} m v^2 + 0$$
$$v^2 = \frac{2\cancel{m}gh}{\cancel{m}} = 2gh$$
$$v = \sqrt{2gh}$$

path does not matter because E is a scalar not a vector which means it has magnitude but not direction

$\vec{P} = mv$

$E = \frac{1}{2}mv^2$

$J = F\Delta t = \Delta P$ | so $\vec{F} = \frac{\Delta P}{\Delta t}$ | & if $\Sigma F = 0$ $\vec{P}$ is constant

Elastic Collisions: $\vec{P}_i = \vec{P}_f$ and $E_i = E_f$

Inelastic collisions: $\vec{P}_i = \vec{P}_f$ but $E_i \neq E_f$

Momentum and Impulse (ch 8)

$\vec{P}_i = \vec{P}_f$ — Momentum ($\vec{P}$) is always conserved in a collision

$J = F \cdot t$

$J = \Delta \vec{P}$

An impulse is a force applied over time and is equal to a change in momentum

So $F\Delta t = \Delta \vec{P}$

$E = F \cdot d$

$\vec{P} = F \cdot t$

remember E is equal to a Force applied ~~over~~ a distance and $\vec{P}$ is equal to a Force applied over time

think about the units & this makes sense

$F \cdot d$ is $kg\frac{m}{s^2} \cdot m = kg\frac{m^2}{s^2}$ which is a Joule

$F \cdot t$ is $kg\frac{m}{s^2} \cdot s = kg\frac{m}{s}$ or $N \cdot s$ which is $\vec{P}$ units

There are 2 types of collisions you will usually see

$\vec{P}_i = \vec{P}_f$ & $E_i = E_f$ — completely elastic - means $\vec{P}$ and E are both conserved and in a typical elastic collision problem both of these will be necessary to solve the problem.

$\vec{P}_i = \vec{P}_f$, $E_i \neq E_f$ * — completely inelastic - means colliding objects stick together and become 1 object. Momentum is conserved as always, but E is not.

Typical set ups for 2 types of collisions above with two objects

*Typically there will be 2 unknowns so both relations will be needed

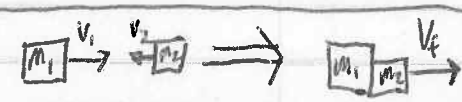
elastic

$\vec{P}_i = \vec{P}_f \rightarrow m_1V_1 + m_2V_2 = m_1V_{1f} + m_2V_{2f}$

$E_i = E_f \rightarrow$

$\frac{1}{2}m_1V_1^2 + \frac{1}{2}m_2V_2^2 = \frac{1}{2}m_1V_{1f}^2 + \frac{1}{2}m_2V_{2f}^2$

inelastic



$\vec{P}_i = \vec{P}_f \rightarrow$

$m_1V_1 + m_2V_2 = (m_1 + m_2)V_f$

Energy is not conserved

$E_i \neq E_f$

Typically has only

$$X_{cm} = \frac{\Sigma m x}{\Sigma m}$$

$I = \Sigma m r^2$ where r is radius from axis of rotation

Center of Mass (ch. 8)

With applied linear Forces, the calculations are done using center of mass as position

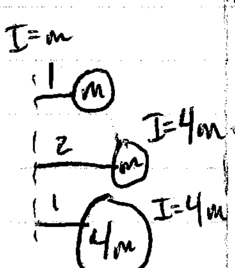
With rotations the rotation occurs about the center of mass

Center of mass is the average position of mass in any object
... or as the book says a "mass-weighted average"

Moment of Inertia (ch. 9)

With linear dynamics only the amount of mass affects the system so "m" is used
In rotational dynamics not only the amount of mass is important, but also how that mass is distributed in space is important. "I" is the quantity that affects a rotational system and is thus used in place of "m" in rotations.

with $F = ma$	"m" determines how much "a" a given force will give to a body with known mass "m". If mass is doubled acceleration is cut in half for equal force
with $\tau = I\alpha$ $\tau = (\Sigma m r^2)\alpha$ doubling radius is equivalent to quadrupling mass	"I" determines how much "α" a given torque gives to body with moment of Inertia "I". - if "m" is doubled α is multiplied by 1/2 as above - if "r" is doubled α is multiplied by 1/4 because r is squared



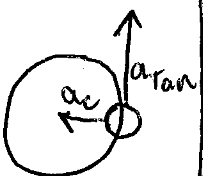
$\boxed{\text{Circumference} = 2\pi r}$

$s = r\theta$

$v = r\omega$

$a = r\alpha \rightarrow$ this is tangential a

$a_c = \frac{v^2}{r} \rightarrow$ this is radial or centripetal a



$\boxed{I \text{ is analog of } m}$ *

$KE = \frac{1}{2} I\omega^2$

$L = I\omega$

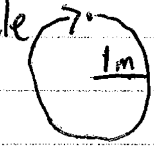
Rotation (ch. 9)

All rotational kinematics are founded on linear kinematics. ~~because radians~~

Radians are used as units of rotation because radians represent a pure number which is the conversion factor to go from distances in a circle to linear distances based on equation for circumference

for example:

if a particle starts here & goes in a circle with $r = 1m$ in $1s$ then



the distance covered by particle is circumference of circle $s = C = 2\pi r$ the velocity is $\frac{\Delta s}{\Delta t}$ so $v = 2\pi(1m)/1s$ or $2\pi \frac{m}{s}$

The angular velocity is $\omega = 2\pi \frac{rad}{s}$ because 1 circle $= 2\pi$ radians

$v = r\omega$ so when $r = 1m$ $v = (1m)(2\pi \frac{rad}{s}) = 2\pi \frac{m}{s}$

if $r = 2m$ $v = (2m)(2\pi \frac{rad}{s}) = 4\pi \frac{m}{s}$

this is because when $r = 2m$ $C = 2\pi r = (2\pi)(2m) = 4\pi m$

Therefore radians are 2π circular conversion #

all equations such as $r\alpha = a$ $r\omega = v$ $r\theta = s$ are merely conversions based on circumference because displacement(s) = circumference $(c) = 2\pi r$

all kinematics equations are equivalent because of for example

$v = v_0 + at$ divide both sides by r gives

$\frac{v}{r} = \frac{v_0}{r} + \frac{at}{r}$ $\frac{v}{r} = \omega$ and $\frac{a}{r} = \alpha$ so

$\omega = \omega_0 + \alpha t$ & same applies to all kinematic equations

$\tau = I\alpha$ | $W_{ork} = \tau \Delta\theta = KE$ | $P_{ower} = \tau\omega$ | so $r \times \vec{p} = I\omega$

$L = I\omega$ | so $\tau \Delta\theta = \frac{1}{2} I\omega^2$ | Power | momentum

$KE = \frac{1}{2} I\omega^2$ | $\Sigma\tau = \frac{\Delta L}{\Delta t}$

$V_{center\ of\ mass} = r\omega$ (without slipping) | $\vec{L} = \vec{r} \times \vec{p} = \vec{r} \times m\vec{v}$

Rotational Dynamics (ch. 10)

Everything is completely analogous to linear dynamics where

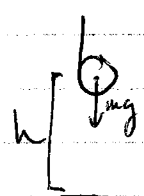
"α" means "analog"

$\theta \,\alpha\, s$	$I \,\alpha\, m$	remember	$s = r\theta$	$I = \frac{\Sigma mr^2}{\Sigma m}$
$\omega \,\alpha\, v$	$\tau \,\alpha\, F$		$v = r\omega$	$\vec{\tau} = \vec{r} \times \vec{F}$
$\alpha \,\alpha\, a$	$\vec{L} \,\alpha\, \vec{p}$		$a = r\alpha$	$\vec{L} = \vec{r} \times \vec{p}$

These are conversion factors between rotation and linear dynamics.

If system is moving linearly and rotationally conversion factors listed above provide bridge between linear and rotational motion

For example let's look at a yo-yo



it starts at rest at height h & is then dropped, E is conserved

$E_i = E_f \qquad E_i = U_i + K_i \qquad E_f = U_f + K_f$

$U_i + K_i = U_f + K_f$ — not moving at top so $K_i = 0$; $h = 0$ at bottom so $U_f = 0$

so $U_i = K_f$ — for K_f it has both linear & rotational E

so

$U_i = \underset{\text{linear } E}{\frac{1}{2}mv^2} + \underset{\text{rotational } E}{\frac{1}{2}I\omega^2}$

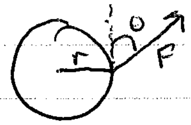
$\boxed{mgh = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2}$ and if it doesn't slip $\boxed{v = r\omega}$

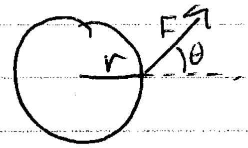
These should be sufficient to solve most problems like this

$\tau = I\alpha$
$\tau = \vec{r} \times \vec{F}$ $\Rightarrow I\alpha = \vec{r} \times \vec{F}$

$\boxed{\text{Torque}}$ (ch. 10)

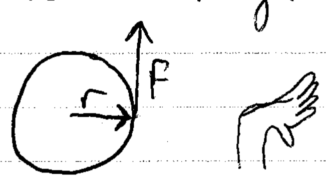
Torque is rotational analog of Force
$\tau = I\alpha$ just like $F = ma$

Torque is <u>cross-product</u> of $\vec{r} \times \vec{F}$. What this means:
① <u>Magnitude</u> is r times perpendicular component of $\vec{F}$
↳ for  $F_\perp = F\cos\theta$ so $\tau = rF\cos\theta$

for  $F_\perp = F\sin\theta$ so $\tau = rF\sin\theta$

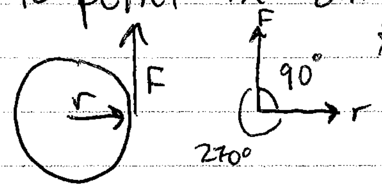
1st vector 2nd vector
$\vec{\tau} = \vec{r} \times \vec{F}$

② <u>Direction</u> is determined by <u>Right Hand Rule</u>
How to do RHR:
① Point fingers of <u>right Hand</u> in direction of 1st vector



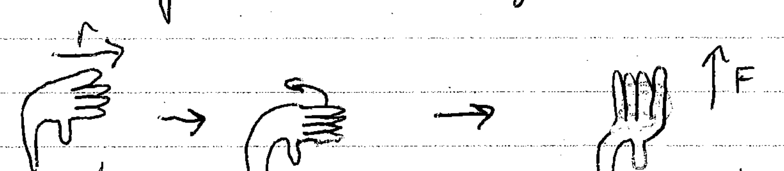
hand "is r-vector" w/ fingers as arrow

② Fingers should be rotated through smallest angle to point in direction of 2nd vector



*use 90° angle not 270° angle

so



hand as r vector → → hand as F-vector Thumb out of p.

③ Direction of thumb is direction of resultant vector.
So above τ is out of the page

* Make sure to use ~~[illegible]~~ right-Hand
i.e. right handers: put your pencil down.

moduli = $\frac{stress}{strain}$

$Y = \frac{F_\perp l_0}{\Delta l A}$

$B = -\frac{P V_0}{\Delta V}$

$S = \frac{F_\parallel h}{\Delta x A}$

Elasticity (ch. 11)

All logic of elasticity is based on stress & strain

stress is the amount of F per area (A) on a body

stress = $\frac{F}{A}$

strain is the deformation an object experiences from a stress

can be $\Delta l / l_0$, $\Delta V / V_0$ or $\Delta x / h$ it is unitless

all moduli follow basic equation of

$$\boxed{\text{moduli} = \frac{\text{stress}}{\text{strain}}}$$

see section 11.4

Tensile → how much something stretches

defined by Young's Modulus (Y)

use only perpendicular component of force

① Tensile stress is Force⊥ over area stress = F/A

② " strain is deformation over total length

-this is a percentage & therefore unitless

so $Y = \frac{stress}{strain} = \frac{F/A}{l} = \frac{F_\perp l_0}{\Delta l A}$

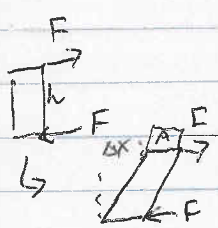
Bulk → how much volume of something changes

defined by Bulk Modulus (B)

① Bulk stress is F/A ; Pressure = F/A ⇒ stress = pressure

② Bulk strain is $\Delta V / V_0$ change in volume over initial volume

so $B = \frac{stress}{strain} = -\frac{P}{\Delta V / V_0} = -\frac{P V_0}{\Delta V}$



Shear → sideways deformation defined by Shear Modulus (S)

① stress is Force parallel over Area $F_\parallel / A$

② strain is sideways deformation over vertical height $\frac{\Delta x}{h}$

so $S = \frac{stress}{strain} = \frac{F_\parallel / A}{\ldots} = \frac{F_\parallel h}{\ldots}$

$\Sigma F_x = 0$
$\Sigma F_y = 0$
$\Sigma F_z = 0$
$\Sigma \tau = 0$

→ definition of Equilibrium

$\boxed{\text{Equilibrium}}$ (ch. 11)

When a body is in equilibrium it means that the sum of all torques and forces is equal to zero. If this were not true the body would be accelerating linearly or rotationally.

Weight of a body can be assumed to be acting at center of mass as center of gravity. This must be used for torque and any forces it may have.

<u>To solve equilibrium problems:</u>

① Break all forces into components
so F becomes F_x, F_y, & F_z
set all sums equal to zero (i.e. $\Sigma F_x = F_{x1} + F_{x2} = 0$ so $F_{x1} = -F_{x2}$)

* see bottom of p. 409, top of p. 410 Blue-box

② Pick convenient reference point for torques. This means use location of one of the forces as axis of rotation (so that r is 0 is the torque "disappears" from one of the forces)

③ set $\Sigma \tau = 0$ and calculate torques circular orbits

$V = \sqrt{\frac{GM}{r}}$

$E = -\frac{Gmm}{2r}$

$T = \frac{2\pi r}{V} = \frac{2\pi r^{3/2}}{\sqrt{Gm_E}}$

elliptical orbits

$\frac{dA}{dt} = \frac{L}{2m}$

Orbits (ch. 12)

$\boxed{\text{Energy is conserved}}$

circular orbits

Force holding object in orbit is both $F_{centripetal}$ and $F_{gravity}$. In other words the centripetal force is provided by gravity.

V is tangential $\rightarrow F_c = ma_c = \frac{mv^2}{r_E}$ $\qquad F_g = \frac{Gmm_E}{r_E^2}$

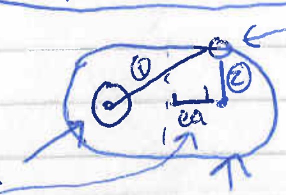
$F_c = F_g$

$\frac{\cancel{m}v^2}{r_E} = \frac{G\cancel{m}m_E}{r_E^2} \longrightarrow V^2 = \frac{Gm_E\cancel{r}}{r^{\cancel{2}}} = \frac{Gm}{r}$

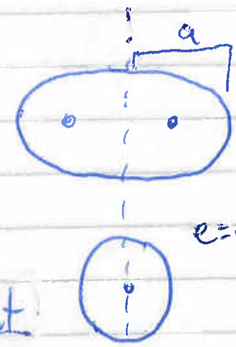
$V = \sqrt{\frac{Gm}{r}}$

p. 452

elliptical orbits



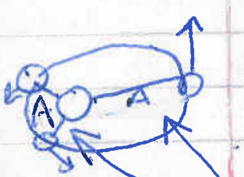
Sun lies at one of foci



e=0 is a circle

sum of 1+2 is constant

distance from axis to foci is eccentricity(e) times the semi-major axis (a) = ea so if e=0 then that distance is 0 and figure is circle



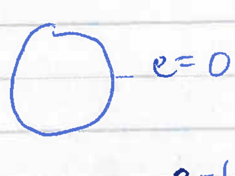
$\frac{dA}{dt}$ is constant. This means the area that the planet carves out in a given time is constant. Both of these areas are the same. This is why planet moves faster at perihelion (edge closer to sun).

$T = \frac{2\pi a^{3/2}}{\sqrt{Gm_s}}$

e is less than or equal to 1 & greater than or equal to 0. This means foci is some percentage of distance to end of major axis

if e=0 it is a circle

if e=1 it is a line



$F = G\frac{m_1 m_2}{r^2}$	$U = \frac{-Gmm}{r}$
so $g = \frac{Gm}{r^2}$	

Keep in mind that problems do not need to give you mass of the earth or sun, all of these values can be found in back of book, or on equation sheet of tests

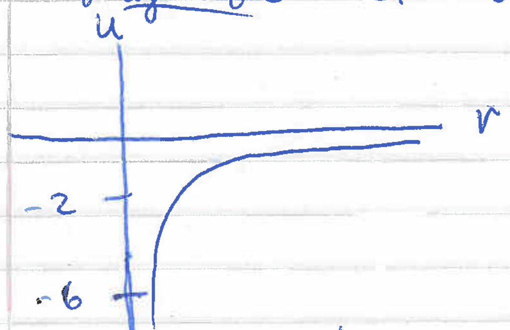
Gravity (ch. 12)

Gravity is the natural tendency of mass to attract mass. It is proportional to amount of mass & inversely proportional to amount of space separating the mass

$$F_g = \cancel{m}g = \frac{G\cancel{m}m_E}{r_E^2} \quad \text{so} \quad g = \frac{Gm_E}{r_E^2}$$

Energy and Gravity

U is negative so one has less potential as magnitude of U increases



one has more potential E at -2 than at -6 even though 6 is more than 2

at $r = \infty$ $U = 0$ and is maximum

at $r = 0$ $U = -\infty$ and is minimum

Escape velocity

V is minimum escape velocity

$E_i = E_f$

$K_i + U_i = K_f + U_f$

$K_i = \frac{1}{2}mv^2 \quad U_i = \frac{-GmM_E}{r_E}$

Find escape v of earth

with minimum escape V_0 → so $K_f = 0$, $V_f = 0$

$U_f = \frac{Gmm_E}{r_f^2} \quad r_f = \infty$ so $U_f = 0$

$$\text{so} \quad \frac{1}{2}mV_{escape}^2 - \frac{GmM_E}{r_E} = 0$$

$$\frac{1}{2}\cancel{m}v^2 = \frac{G\cancel{m}m_E}{r_E}$$

$$V_{escape} = \sqrt{\frac{2Gm_E}{r_E}}$$

This seems weird but works because of the strange nature of $U_{gravity}$

Simple pendulums

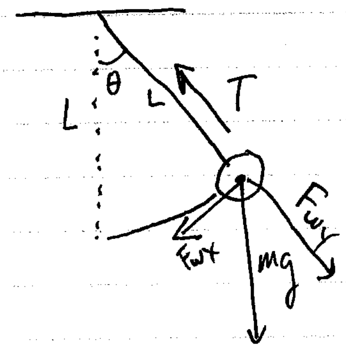
$\omega=\sqrt{\frac{k}{m}}=\sqrt{\frac{g}{L}}$	restoring F: $F_\theta=-mg\sin\theta$
$f=\frac{\omega}{2\pi}=\frac{1}{2\pi}\sqrt{\frac{g}{L}}$	when θ is small $F_\theta=-\frac{mg}{L}x=-mg\theta$
$T=\frac{2\pi}{\omega}=\frac{1}{f}=2\pi\sqrt{\frac{L}{g}}$	

Pendulums (ch. 13)

see p.495

Simple pendulums

i.e. weight on a string — all mass concentrated at a single point



$F_\theta=0$ when $\theta=0$

$F_{wx}=F_\theta=-mg\sin\theta$ This is restoring Force

$F_{wy}=mg\cos\theta$

Restoring Force is always negative because it always points inwards from direction of displacement

Tension would be $F_{wy}+F_{centripetal}$ $T=F_{wy}+F_c$

F_c is not constant because V is not constant

$F_{wy}=mg\cos\theta$ and θ is not constant so neither is F_{wy}

Simple pendulums undergo simple Harmonic motion where $\omega=\sqrt{\frac{k}{m}}=\sqrt{\frac{g}{L}}$ $\omega=\sqrt{\frac{g}{L}}$ meaning period is independent of mass

in this case $k=\frac{mg}{L}$ so where $\omega=\sqrt{\frac{k}{m}}=\sqrt{\frac{mg/L}{m}}=\sqrt{\frac{g}{L}}$ m divides out

ω is only dependent on $g\left(\frac{Gm}{r^2}\right)$ and L

- So a pendulum with known L can be used to test unknown strength of g (or vice versa)

Resonance (ch. 13) p. 502

All physical systems oscillate at a natural frequency (for pendulums it is $\omega=\sqrt{\frac{g}{L}}$). If a driving frequency matches the natural frequency of a system this is resonance and maximum amplitude (A) increases very fast.

with driving force $A=\frac{F_{max}}{\sqrt{(k-m\omega_d^2)^2+b^2\omega_d^2}}$ where ω_d is driving f

one can see A is at a maximum when denominator is at a minimum. This occurs when $\boxed{\omega_d=\sqrt{\frac{k}{m}}}$

when $\omega_d=\sqrt{\frac{k}{m}}$ then $A=\frac{F_{max}}{(k-m(\sqrt{\frac{k}{m}})^2)^2+b^2\omega_d^2}=\frac{F_{max}}{b^2\omega_d^2}$

ie for a spring $\omega=\sqrt{\frac{k}{m}}$ when driving f also $=\sqrt{\frac{k}{m}}=$ natural frequency Force reinforces

$$E = \frac{1}{2}mv^2 + \frac{1}{2}kx^2 = \frac{1}{2}kA^2 = \text{constant}$$

$\boxed{\text{Simple Harmonic Motion}}$ (ch. 13) $\boxed{\text{Part Two}}$

continued from last page

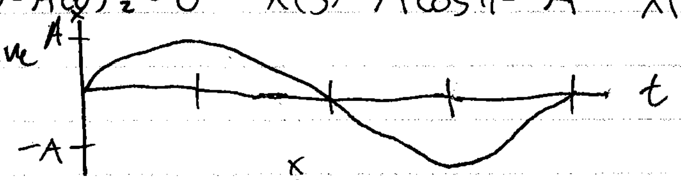
ϕ is phase angle, it is how much graph is shifted left or right

look back at grdp s on last page

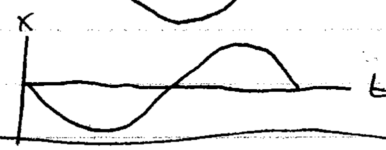
if $\phi = -\frac{\pi}{2}$ then $x(0) = A\cos -\frac{\pi}{2} = 0$ $x(1) = A\cos 0 = A$

$x(2) = A\cos\frac{\pi}{2} = 0$ $x(3) = A\cos\pi = -A$ $x(4) = A\cos\frac{3\pi}{2} = 0$

This is a sine wave



if $\phi = \frac{\pi}{2}$ it is reverse sine wave



Energy in SHM

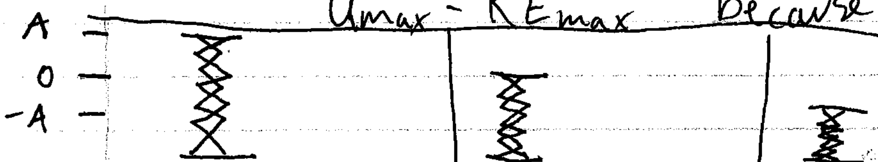
U=potential energy
KE=kinetic energy

Energy is conserved

E fluctuates between U (which is $\frac{1}{2}kx^2$) & KE ($\frac{1}{2}mv^2$)

- when $v=0$ (at maximum displacement) x is at maximum
  - $KE = \frac{1}{2}m(0)^2 = 0$ and potential energy is maximum
- when $x=0$ (at equilibrium point) v is at a maximum
  - $U = \frac{1}{2}k(0)^2 = 0$ and kinetic energy is maximum

$U_{max} = KE_{max}$ because E is conserved



U is max $= \frac{1}{2}kx^2 = \frac{1}{2}kA^2$ KE=0	KE is max $= \frac{1}{2}mv^2 = \frac{1}{2}kA^2$ U=0	U is max $= \frac{1}{2}kx^2 = \frac{1}{2}kA^2$ KE=0	u is still positive when $x=-A$ because $U = \frac{1}{2}kx^2$ $-A$ is squared & thus positive

NOT for acceleration const. kinematic equations

p. 484
p. 485

$x(t) = A\cos(\omega t + \phi)$ | $V_x(t) = \frac{dx}{dt} = -\omega A \sin(\omega t + \phi)$ | $F = -kx$

$A = \sqrt{x_0^2 + \frac{V_{0x}^2}{\omega^2}}$ 14.22 | $a_x(t) = \frac{dv}{dt} = -\omega^2 A\cos(\omega t + \phi)$ | $E = \frac{1}{2}kx^2$

$\phi = \tan^{-1}\left(-\frac{V_{0x}}{\omega x_0}\right)$ | $\omega = \sqrt{\frac{k}{m}}$ | $f = \frac{\omega}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$ | $T = \frac{1}{f} = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{m}{k}}$

Simple Harmonic Motion (ch. 13) Part One

k = spring constant

$F = -kx$ so F is bigger when displacement (x) is bigger. Force is stronger when further from equilibrium.

Understanding $x(t) = A\cos(\omega t + \phi)$

① A is maximum displacement (Amplitude)

② $\omega t = \theta$: just like vt = displacement ωt = angle of displacement

③ ϕ is the phase angle and is only the starting displacement angle (θ_0) if $\phi = -\frac{\pi}{2}$ graph is sine wave rather than cosine

④ $x(t)$ is the displacement at any given time t
- it will always be some percentage of the maximum displacement (A) that is why $-1 \leq \cos(\omega t + \phi) \leq 1$

How the equation works

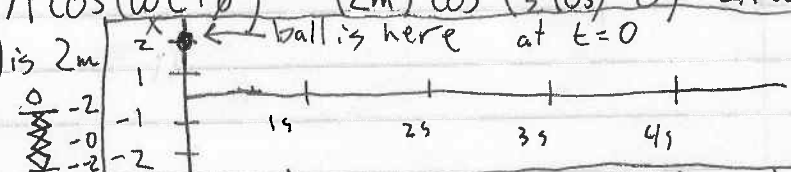
a ball on the end of a spring

imagine a situation $x(t) = A\cos\theta$ where $\theta = \omega t + \phi$
let $\phi = 0$ rad and $A = 2$ m and $\omega = \frac{\pi}{2}$ rad/s

① at $t = 0$ $x(t) = A\cos(\omega t + \phi) = (2m)\cos\left(\frac{\pi}{2}(0s) + 0\right) = 2m\cos 0$

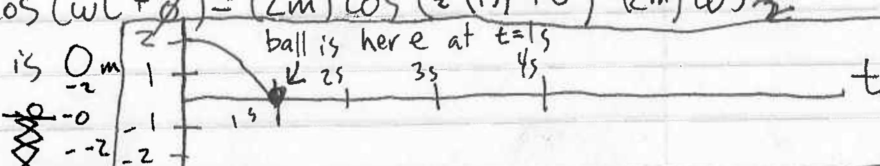
$(2m)\cos 0 = 2m$ so $x(0)$ is $2m$

spring is aligned w/ y-axis

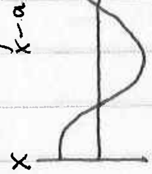


② at $t = 1s$ $x(t) = A\cos(\omega t + \phi) = (2m)\cos\left(\frac{\pi}{2}(1s) + 0\right) = (2m)\cos\frac{\pi}{2}$

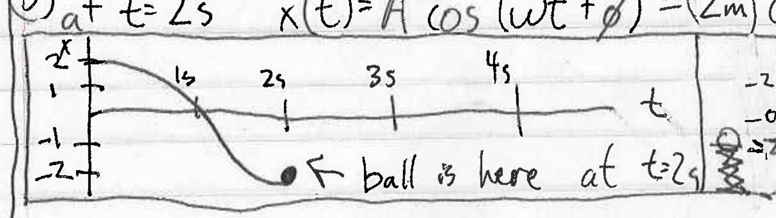
$(2m)\cos\frac{\pi}{2} = 0m$ so $x(1)$ is $0m$



Axes: y-axis is displacement (x)
x-axis is time (t)

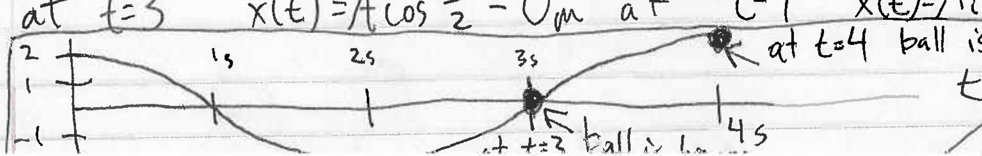


③ at $t = 2s$ $x(t) = A\cos(\omega t + \phi) = (2m)\cos\left(\left(\frac{\pi}{2}\frac{rad}{s}\right)(2s) + 0\right) = 2m\cos\pi = -2m$



we start to see cosine wave forming through time. if ϕ were $-\frac{\pi}{2}$ $x(2)$ would be $2\cos\frac{\pi}{2} = 0$ & ⇒ a sine wave.

④ at $t = 3$ $x(t) = A\cos\frac{3\pi}{2} = 0m$ at $t = 4$ $x(t) = A\cos 2\pi = 2m$



if $\phi = -\frac{\pi}{2}$ all values are shifted to left & sine wave results

Physical pendulums → $\omega = \sqrt{\frac{mgd}{I}}$ ⇒ remember $I = \Sigma mr^2$ so $\sqrt{\frac{\cancel{m}gd}{\cancel{m}r^2}}$ is still independent of m

center of gravity = center of mass

p. 497

$\boxed{\text{Pendulums}}$ (ch. 13)

physical pendulums don't have all mass concentrated at a single point like simple pendulums

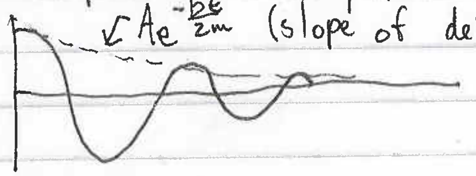
→ - calculations are done using the center of gravity as the point where all mass is concentrated

$\omega = \sqrt{\frac{mgd}{I}}$ instead of $\sqrt{\frac{g}{L}}$

$\boxed{\text{Damped Oscillations}}$

Max. Displacement Amplitude (A) gets smaller & smaller & approaches 0

$Ae^{\frac{-bt}{2m}}$ (slope of descending A_{max})



new equation is $Ae^{\frac{-bt}{2m}}\cos(\omega t + \phi)$

only difference from SHM equation $x(t) = A\cos(\omega t + \phi)$ is A is not constant but varies with time

$e^0 = 1$ $A(t) = Ae^{\frac{-bt}{2m}}$ so at $t=0$ $A = Ae^0 = A$

but as t increases $e^{\frac{-bt}{2m}}$ approaches 0

A(t) is therefore a fraction [illegible] nal A_{max}

so not only is x(t) varying with time but so is maximum displacement (A)

A Damped Oscillation is where some force such as friction or drag is taking energy out of the oscillating system so the maximum displacement is getting smaller and smaller