

So also  
 $F \cdot s = \Delta E$   
 $\& F = \frac{\Delta E}{\Delta s}$

$$K_1 + U_1 + W_{\text{conservative}} = K_2 + U_2$$

$$U_{\text{spring}} = \frac{1}{2} k x^2$$

$$W = F \cdot s = \Delta E \text{ (can be } U \text{ or } K)$$

$$KE = \frac{1}{2} m v^2$$

$$P = \frac{\Delta W}{\Delta t} = \vec{F} \cdot \vec{v}$$

$$U_{\text{gravity}} = mgh$$

## Conservation of Energy (ch. 6 & 7)

If only conservative forces (i.e. gravity or springs) are acting in a system all E is conserved and constant. So  $\Sigma E = KE + U$  and  $E_0 = E_f$  initial E is same as final E

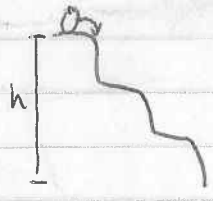
see bottom of p. 261 & p. 262 \*

Energy that is not conserved is lost through forces that take E out of the system as heat, sound, or some other unrecoverable quantity.

Friction, drag both are forces that do negative work in both directions and are not conservative F.

Work is the amount of E a F puts into the system. It can be recovered (the E) as long as the force is conservative.

Energy is a scalar and so In equations:



path does not matter.  
 both balls start with  $U = mgh$   
 and both end with same  $\vec{v}$  where  $\vec{v} = \sqrt{2gh}$  here's why

$$K_i = 0$$

$$U_f = 0$$

$$E_0 = E_f$$

$$K_1 + U_1 = K_2 + U_2$$

$$0 + mgh = \frac{1}{2} m v^2 + 0$$

$$\sqrt{2} = \frac{2mgh}{m} = 2gh$$

$$v = \sqrt{2gh}$$

path does not matter because E is a scalar not a vector which means it has magnitude but not direction

$$\vec{p} = m\vec{v}$$

$$E = \frac{1}{2}mv^2$$

$$J = F\Delta t = \Delta \vec{p} \quad \text{so} \quad \vec{F} = \frac{\Delta \vec{p}}{\Delta t} \quad \& \quad \text{if } \sum \vec{F} = 0 \quad \vec{p} \text{ is constant}$$

Elastic collisions:  $\vec{p}_i = \vec{p}_f$  and  $E_i = E_f$

Inelastic collisions:  $\vec{p}_i = \vec{p}_f$  but  $E_i \neq E_f$

## Momentum and Impulse (ch 8)

$\vec{p}_i = \vec{p}_f$  Momentum ( $\vec{p}$ ) is always conserved in a collision

$J = F \cdot t$   
 $J = \Delta \vec{p}$   
 An impulse is a force applied over time and is equal to a change in momentum  
 so  $F\Delta t = \Delta \vec{p}$

$E = F \cdot d$   
 $\vec{p} = F \cdot t$   
 remember  $E$  is equal to a Force applied over a distance and  $\vec{p}$  is equal to a Force applied over time  
 think about the units & this makes sense

$F \cdot d$  is  $\text{kg} \frac{\text{m}}{\text{s}^2} \cdot \text{m} = \text{kg} \frac{\text{m}^2}{\text{s}^2}$  which is a Joule

$F \cdot t$  is  $\text{kg} \frac{\text{m}}{\text{s}^2} \cdot \text{s} = \text{kg} \frac{\text{m}}{\text{s}}$  or  $\text{N} \cdot \text{s}$  which is  $\vec{p}$  units

$$\vec{p}_i = \vec{p}_f$$

$$E_i = E_f$$

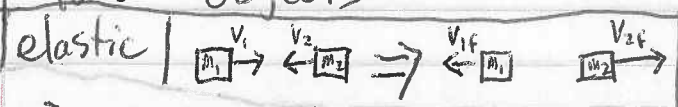
There are 2 types of collisions you will <sup>usually</sup> see  
completely elastic - means  $\vec{p}$  and  $E$  are both conserved and in a typical elastic collision problem both of these will be necessary to solve the problem.

$$\vec{p}_i = \vec{p}_f$$

$$E_i \neq E_f$$

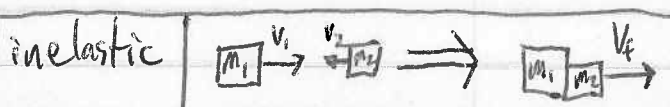
completely inelastic - means colliding objects stick together and become 1 object. Momentum is conserved as always, but  $E$  is not.

Typical setups for 2 types of collisions above with two objects



$$\vec{p}_i = \vec{p}_f \Rightarrow m_1 v_1 + m_2 v_2 = m_1 v_{1f} + m_2 v_{2f}$$

$$E_i = E_f \Rightarrow \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2 = \frac{1}{2} m_1 v_{1f}^2 + \frac{1}{2} m_2 v_{2f}^2$$



$$\vec{p}_i = \vec{p}_f \Rightarrow m_1 v_1 + m_2 v_2 = (m_1 + m_2) v_f$$

Energy is not conserved  
 $E_i \neq E_f$  Typically has only 1 equation

\* Typically there will be 2 unknowns so both relations will be needed

$$X_{cm} = \frac{\sum mX}{\sum m}$$

$I = \sum mr^2$  where  $r$  is radius from axis of rotation

### Center of Mass (ch. 8)

With applied linear forces, the calculations are done using center of mass as position

With rotations the rotation occurs about the center of mass

Center of mass is the average position of mass in any object  
 ... or as the book says a "mass-weighted average"

### Moment of Inertia (ch. 9)

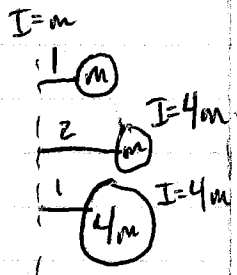
With linear dynamics only the amount of mass affects the system so " $m$ " is used  
 In rotational dynamics not only the amount of mass is important, but also how that mass is distributed in space is important. " $I$ " is the quantity that affects a rotational system and is thus used in place of " $m$ " in rotations.

with  $F=ma$

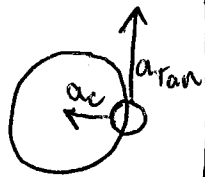
" $m$ " determines how much " $a$ " a given force will give to a body with known mass " $m$ ". If mass is doubled acceleration is cut in half for equal force

with  $\tau = I\alpha$   
 $\tau = (\sum mr^2)\alpha$

" $I$ " determines how much " $\alpha$ " a given torque gives to body with moment of Inertia " $I$ ".  
 - if " $m$ " is doubled  $\alpha$  is multiplied by  $1/2$  as above  
 - if " $r$ " is doubled  $\alpha$  is multiplied by  $1/4$  because  $r$  is squared



→ doubling radius is equivalent to quadrupling mass



Circumference = $2\pi r$
$s = r\theta$
$v = r\omega$
$a = r\alpha \rightarrow$ this is tangential $a$
$a_c = \frac{v^2}{r} \rightarrow$ this is radial or centripetal $a$

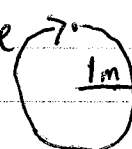
$I$  is analog of  $m$  \*

$KE = \frac{1}{2} I \omega^2$
$L = I \omega$

## Rotation (ch. 9)

All rotational kinematics are founded on linear kinematics. ~~because radians~~

Radians are used as units of rotation because radians represent a pure number which is the conversion factor to go from distances in a circle to linear distances based on equation for circumference for example:

if a particle starts <sup>here</sup>  $\rightarrow$  & goes in a circle  with  $r=1m$  in 1s then

the distance covered by particle is circumference of circle  
 $s = C = 2\pi r$  the velocity is  $\frac{s}{t}$  so  $v = 2\pi(1m)/1s$   
 or  $2\pi \frac{m}{s}$

The angular velocity is  $\omega = 2\pi \frac{rad}{s}$  because 1 circle =  $2\pi$  radians  
 $v = r\omega$  so when  $r=1m$   $v = (1m)(2\pi \frac{rad}{s}) = 2\pi \frac{m}{s}$   
 if  $r=2m$   $v = (2m)(2\pi \frac{rad}{s}) = 4\pi \frac{m}{s}$

this is because when  $r=2m$   $C = 2\pi r = (2\pi)(2m) = 4\pi m$

Therefore radians are  $2\pi$  circular conversion #  
 all equations such as  $r\alpha = a$   $r\omega = v$   $r\theta = s$  are merely conversions based on circumference because  
 displacement( $s$ ) = circumference ( $C$ ) =  $2\pi r$

all kinematics equations are equivalent because of  $\uparrow$   
 for example

$v = v_0 + at$  divide both sides by  $r$  gives  $\downarrow$   
 $\frac{v}{r} = \frac{v_0}{r} + \frac{at}{r}$   $\frac{v}{r} = \omega$  and  $\frac{a}{r} = \alpha$  so  $\downarrow$

$\omega = \omega_0 + \alpha t$  & same applies to all kinematic equations



$\vec{\tau} = I\alpha$	$W_{\text{ork}} = \tau \Delta\theta = KE$	$P_{\text{ower}} = \tau \omega$	$\text{so } r \times \vec{p} = I\omega$
$\vec{L} = I\omega$	$\text{so } \tau \Delta\theta = \frac{1}{2} I \omega^2$	↑ Power	
$KE = \frac{1}{2} I \omega^2$	$\sum \tau = \frac{\Delta L}{\Delta t}$	↓ momentum	
$V_{\text{center of mass}} = r\omega$ (without slipping)		$\vec{L} = \vec{r} \times \vec{p} = \vec{r} \times m\vec{v}$	

## Rotational Dynamics (ch. 10)

Everything is completely analogous to linear dynamics where

"x means analog"

$\theta \propto s$	$I \propto m$	remember	$s = r\theta$	$I = \frac{\sum mr^2}{\sum m}$
$\omega \propto v$	$\tau \propto F$		$v = r\omega$	$\tau = \vec{r} \times \vec{F}$
$\alpha \propto a$	$\vec{L} \propto \vec{p}$		$a = r\alpha$	$\vec{L} = \vec{r} \times \vec{p}$

These are conversion factors between rotation and linear dynamics.

If system is moving linearly and rotationally conversion factors listed above provide bridge between linear and rotational motion. For example let's look at a yo-yo



it starts at rest at height  $h$  & is then dropped,  $E$  is conserved

$$E_i = E_f \quad E_i = U_i + K_i \quad E_f = U_f + K_f$$

$$U_i + K_i = U_f + K_f \quad \text{not moving at top so } K_i = 0$$

$$h = 0 \text{ at bottom so } U_f = 0$$

$$\text{so } U_i = K_f$$

$$U_i = \frac{1}{2} m v^2 + \frac{1}{2} I \omega^2$$

↑ linear E      ↑ rotational E

for  $K_f$  it has both linear & rotational E

$$mgh = \frac{1}{2} m v^2 + \frac{1}{2} I \omega^2$$

and if it doesn't slip

$$v = r\omega$$

These should be sufficient to solve most problems like this.

$$\tau = I\alpha \Rightarrow I\alpha = \vec{r} \times \vec{F}$$

$$\tau = \vec{r} \times \vec{F}$$

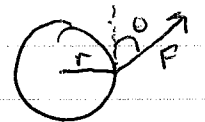
## Torque (ch. 10)

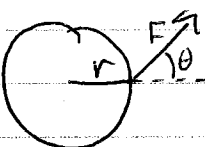
Torque is rotational analog of Force

$$\tau = I\alpha \text{ just like } F = ma$$

Torque is cross-product of  $\vec{r} \times \vec{F}$ . What this means ↓

① Magnitude is  $r$  times perpendicular component of  $\vec{F}$

↳ for   $F_L = F \cos \theta$  so  $\tau = r F \cos \theta$

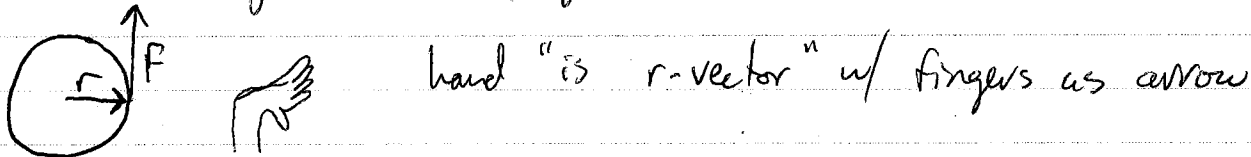
for   $F_L = F \sin \theta$  so  $\tau = r F \sin \theta$

1st vector  $\vec{r}$   
2nd vector  $\vec{F}$   
 $\vec{\tau} = \vec{r} \times \vec{F}$

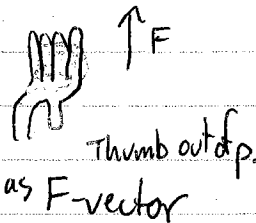
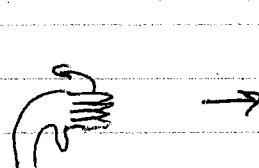
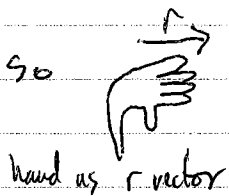
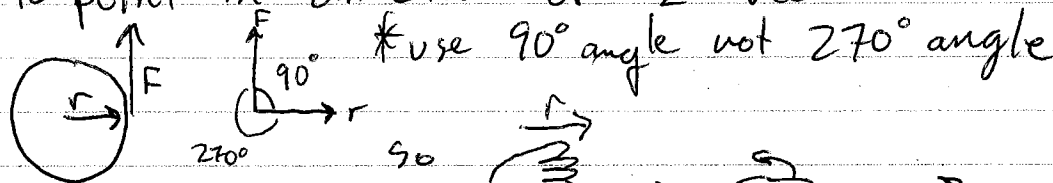
② Direction is determined by Right Hand Rule

How to do RHR:

① Point fingers of right hand in direction of 1<sup>st</sup> vector



② Fingers should be rotated through smallest angle to point in direction of 2<sup>nd</sup> vector



③ Direction of thumb is direction of resultant vector.

So above  $\tau$  is out of the page

\* Make sure to use right-hand  
i.e. right handers: put your pencil down.

$$\text{moduli} = \frac{\text{stress}}{\text{strain}}$$

$$Y = \frac{F_{\perp} l_0}{\Delta l A}$$

$$B = -\frac{P V_0}{\Delta V}$$

$$S = \frac{F_{\parallel} h}{\Delta x A}$$

## Elasticity (ch. 11)

All logic of elasticity is based on stress & strain

stress is the amount of  $F$  per area ( $A$ ) on a body  
 $\text{stress} = \frac{F}{A}$

strain is the deformation an object experiences from a stress  
 can be  $\Delta l / l_0$ ,  $\Delta V / V_0$  or  $\Delta x / h$  it is unitless

all moduli follow basic equation of

$$\text{moduli} = \frac{\text{stress}}{\text{strain}}$$

see section 11.4

use only perpendicular component of force

Tensile → how much something stretches  
 defined by Young's Modulus ( $Y$ )

① Tensile stress is Force  $\perp$  over area

$$\text{stress} = \frac{F}{A}$$

② " strain is deformation over total length

-this is a percentage & therefore unitless

$$\text{so } Y = \frac{\text{stress}}{\text{strain}} = \frac{F/A}{l/l_0} = \frac{F l_0}{\Delta l A}$$

Bulk → how much volume of something changes  
 defined by Bulk Modulus ( $B$ )

① Bulk stress is  $F/A$ ; Pressure =  $F/A \Rightarrow$  stress = pressure

② Bulk strain is  $\Delta V / V_0$ , change in volume over initial volume

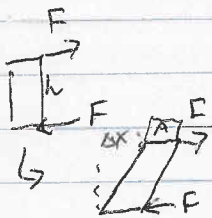
$$\text{so } B = \frac{\text{stress}}{\text{strain}} = -\frac{P}{\Delta V / V_0} = -\frac{P V_0}{\Delta V}$$

Shear → sideways deformation defined by Shear Modulus ( $S$ )

① stress is Force parallel over Area  $F_{\parallel}/A$

② strain is sideways deformation over vertical height  $\frac{\Delta x}{h}$

$$\text{so } S = \frac{\text{stress}}{\text{strain}} = \frac{F_{\parallel}/A}{\Delta x/h} = \frac{F_{\parallel} h}{\Delta x A}$$



$$\begin{aligned} \sum F_x &= 0 \\ \sum F_y &= 0 \\ \sum F_z &= 0 \\ \sum \tau &= 0 \end{aligned} \rightarrow \text{definition of Equilibrium}$$

## Equilibrium (ch. 11)

When a body is in equilibrium it means that the sum of all torques and forces is equal to zero. If this were not true the body would be accelerating linearly or rotationally.

Weight of a body can be assumed to be acting at center of mass as center of gravity. This must be used for torque and only forces it may have.

To solve equilibrium problems:

① Break all forces into components

so  $F$  becomes  $F_x, F_y, \& F_z$   
 set all sum equal to zero (ie.  $\sum F_x = F_{x1} + F_{x2} = 0$  so  $F_{x1} = -F_{x2}$ )

② Pick convenient reference point for torques. This means use location of one of the forces as axis of rotation (so that  $r$  is 0 is the torque "disappears" from one of the forces)

③ set  $\sum \tau = 0$  and calculate torques

\* see bottom of  
 P. 409, top of P. 410  
 Blue-box



