DAY 1

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GETTING STARTED WITH THE GRE

1. GRE Prelude

Unraveling the GRE

The most unique element of the GRE is that it is a Computer Adaptive Test (CAT). This means you will only see one question at a time as you go through the exam, and you ***must*** *answer* that question before moving to the next question. The first question of any section is intermediate in difficulty. Where you go from there is determined by how you answer: if you get it correct the next question is harder; if you get it wrong the next question is easier. The same method pertains to every subsequent question. As such, taking the GRE is like one big choose-your-own-adventure novel, and where you end up is what determines your score.

Your score is not based on the number of questions right or wrong but rather where you "end up" in the complicated matrix of questions. Each score (640, 720, et. al.) has a set of questions corresponding to that level of difficulty.

Your score is equivalent to the level of questions in which you get 50% right and 50% wrong. This has a few non-obvious implications, mainly that early questions are much more important than the final questions. **So, you should spend extra time on the first few questions, even at the cost of insufficient time later**.

If, early in the test, you flub a 600-level question, and your "objective score level" is at 720, you have missed a question the GRE thinks you have a near-100% chance of answering correctly. Thus this is equivalent to missing a whole question. However, if you are at a "720 objective score level" by the end of the test, you are only receiving questions you have a 50% chance of answering correctly. Thus, a wrong answer is tantamount to missing half a question only, based on their statistical rubric. **In short, spend extra time on the first 10 questions, triple-checking each one, while double-checking all questions from 10-20.**

If you've ever taken the GRE before, you'll notice this method is antithetical to your natural inclination to move quickly through early questions as the clock ticks down in the upper right corner of your computer screen. Bit time paranoia can have a significant negative impact on your score. Rushing through 3 early questions is equivalent to putting 3 questions on the line, whereas rushing through the last 3 is equivalent to putting only 1.5 questions on the line. In addition, poor results on the first 10 questions will create a score ceiling that no amount of later success can transcend.

Focusing on the Here and Now

As previously mentioned, you must answer each question before advancing, rather than skipping and going back to check on past problems, which is typically a great test-taking strategy. This means you'll need to develop strategies for optimal guessing (we'll talk about this more later in the course) for those tricky and frustrating problems that might otherwise gobble your time.

The format of the GRE CAT should never leave your mind as you prepare. When you are doing paper practice tests, make sure not to skip ahead or come back to old questions: take each question one at a time, just as you will be forced to do on the CAT version.

Learning to reconcile yourself with the tyranny of time is especially important when taking GRE practice tests. Make sure to always time yourself as you develop a sense for the duration of the test as well as each question. Specific pacing, however, should be the last thing to develop in your GRE strategy. Take the time, first—until the week before your test—to master all constituent material! The goal is not to become hyper-conscious of time: it's to internalize that "feel for time" as you go so it never reaches the forefront of your thoughts or causes anxiety. Focus on understanding the process, and getting all the questions right in your early preparation, and only once you have the test down should you focus on trying to increase your speed; if you focus too much on speed too early, you risk not gaining a full understanding of the material and process.

Anxiety and Letting Go

Stress is an inextricable part of our psychology. But, as you are likely aware, it can have a deleterious impact on test results: abstract problem-solving tasks, such as those on the GRE, require calm, conscious reasoning carried out by specific "mental programs." Anxiety inactivates these very mechanisms. Thus, it is imperative to manage your anxiety to maximize your success.

So here's one thing you really need to know that should help you relax: YOU WILL MISS QUESTIONS ON THE GRE. The test is designed to be as difficult for each test-taker as possible. Remember, your goal is to get to the highest level where you are getting 50% correct. Obviously, 50% correct requires you to get 50% wrong. In fact, you can miss questions (that's right: question**s**, plural!) and still get an 800, so don't let missing a few get to you. As a corollary, you should not try to assess your progress while taking the test. This only distracts from the task at hand. You'll have your score immediately upon completion, so try to learn how to focus all of your attention on answering each question as it comes.

Another major factor in relaxing is preparation: a track record of success will minimize panic. Good news: that preparation is exactly what you are reading this for! That said, despite the understandable desire for last minute cramming, the night before the test should not involve studying, as this will usually not help you minimize stress on test day. You've already worked hard, so take it easy with an early dinner and a movie.

What the GRE Tests, and Why Grad Programs Care (or not, depending on the program)

There are a number of controversial issues surrounding standardized testing, including differences across genders and cultures. So why do graduate programs still use GRE scores as an important factor in evaluating your application? The answer is for its predictive ability. GRE scores are correlated with future performance, so admissions committees still ask for them as one more piece of information to try and predict your success as a grad student. This reasoning leads to a few interesting inferences:

(1) Different programs care about different aspects of your score, and some don't really care at all. If you are applying for a physics program, you had better nail the quantitative section, but your verbal score is likely not especially important; if you are applying to a literature program the reverse is likely true. Furthermore, everyone takes the same tests, and admissions committees realize the literature people are competing against the physics folks in the quantitative, and vice versa for the verbal.

(2) The GRE is only one piece of information that programs use to evaluate your application, and mediocre scores can often be offset by grades and experience. A great score will only make your application that much stronger, but an average score is not the end of the world (or your chances at admission).

(3) Because all that matters is how well the score predicts your future graduate performance, it is probably drawing on a number of important characteristics, not just your mathematical and verbal talents. For example, a large part of succeeding as a graduate student is having the discipline and ability to dedicate your time and effort to academic work. The GRE definitely tests preparation to some extent. Your score doesn't

just illuminate your verbal or mathematical acumen: it also shows how much you prepared for its unique format. In addition, one of the greatest predictors of success as a graduate student is the ability to problem solve. This is why we say the GRE is a problem-solving test, not just a math or vocabulary test. Some questions seem specifically designed to probe exactly this, such as verbal questions in which you are clearly not expected to know the word but are still expected to solve the question.

(4) Lastly, the verbal and quantitative sections are comparative tests (also called norm-referenced tests), placing you in context of everyone else taking the test. Your percentile is much more important than your raw score, and the raw scores for verbal and quantitative sections map onto very different percentiles. On the other hand, the analytical writing section is what is referred to as a criterion-referenced test. This means that it is essentially just a test that verifies you meet some criterion (like a driving test), and does not compare you to other people who took the test. All you need to do is show that you can write a focused 5-paragraph essay like you did in high school.

The New Questions

Recently ETS has started adding new question types that you may have heard about. Most students are stressed about these, but you need not worry for a couple of reasons. First, they don't count towards your score… at least not this year. Second, you will not get more than one new question, if any at all. Third, they are not too different, and you will certainly have the skills you need to answer them. For instance, you'll learn all the math necessary to tackle those new-fangled "fill in" questions that don't present any multiple choice options. In short: don't sweat the new stuff.

2. The Veritas Approach

The GRE is a Problem-Solving Test, not a Math or Vocabulary Test

As mentioned above, you should approach the GRE as a problem-solving test, and not as a math or vocabulary test. You will need strong math skills and you will need to memorize a lot of vocabulary, but these are merely tools you will use to demonstrate your problem-solving abilities in verbal and mathematical domains. You will often need to use a much different approach to solving problems than you would in any other test setting. Plus, the problem-solving approach can make the GRE a competitive game between you and the test-makers, which is much more fun than a test.

Good news: the math is basic! If you encounter some mathematical concept that you are rusty on, and struggling with, use the Internet: there are a ton of great resources online. For the most part, however, all you need can be found within these pages.

When you come up against questions that leave you dumbfounded (and remember, you will and it's nothing to worry about!), try to analyze the structure of the problem to see if there is some other non-obvious way to solve it. For example, eliminate any sucker answers*:* answers that are both tempting for a variety of reasons and are clearly wrong. They will be pointed out numerous times throughout this course. Another trick is to use the structure of the question to eliminate impossible answers (such as eliminating answers that don't have opposites in antonym questions). Because the GRE is a problem-solving test, often all it takes is a flexible approach that allows you to reframe the situation into one that is easily manageable. If you get too stuck on one approach to a question (typically the standard approach apart from the GRE), you will likely miss approaches that would make the beguiling question simple.

The Tao of the GRE™

You are going to be learning a lot of techniques, methods, formulas, words, and facts, but real success comes from putting it all together for each specific problem. The emphasis of this course will not be on techniques themselves so much as learning when and how to apply them. We call the approach the *Tao of the GRE™*, for its mantra about tuning into questions and discovering their internal logic. Okay…so you don't quite have to "be the test," but you should be aware of the logic of the test-makers.

When you first start doing GRE problems, they will likely seem foreign and bizarre. You might get frustrated and think, "How can we be expected to figure that out?" or "How can they possibly say that is the only correct answer?" Don't fret: with familiarity, you'll soon be getting those *Eureka!* moments as expedient paths to solutions appear. For instance, if calculations seem lengthy, you'll soon be estimating your way to an answer. If a word seems excessively confounding, you'll be breaking it up into roots. You'll even start noticing 30-60-90 triangles in the most unlikely of places.

Quantitative Strategies: the Mathematical Tool Belt™

Clearly the Tao of the test differs for different domains; for the quantitative section the Tao entails employing your mathematical tool belt. Throughout the course you will learn a number of different things you need to know: the formula for the area of a circle, square root laws, how to factor, etc. Think of these as the tools in your belt. The trick is to know what tools to apply for what problems and when. Once you are familiar with the tools and the logic behind the questions, you will see that *the problems actually tell you which tools you will need!* Once you become fluent in GREse you will notice that the information that is already there in the question may be your greatest ally in figuring out which tools to employ for the problem at hand.

Verbal Strategies: the Analytical Checklist™

Applying the Tao to the verbal section is different. Rather than a toolbelt full of mathematical formulae, you will engage the questions with two types of complementary checklists: one for figuring out new words and another for the each of the specific question types (analogies, antonyms, sentence-completion, and reading comprehension). These checklists are hierarchically organized from the best approach down, such that you will always try to employ your best strategy first, and only if you can't move on to the second best, the third best, and so on. There are also general strategies to help in any situation. In short, you'll never feel lost at any point because by going through the checklist, you can be confident that you are employing the best possible technique to any given problem.

THE QUANTITATIVE SECTION

3. Quantitative Section Overview

Getting started. The quantitative section of the GRE tests your problem-solving abilities and basic mathematical competence. All of the mathematical concepts you will need for the GRE are covered in this course: make sure you know them all by heart. But first, here is a list of things of which you should be aware:

- You will *NOT* be able to use a calculator on the test, but you will have scratch paper. This means estimation will be one of your most valuable tools. In your preparation, be sure to do your calculations on a separate sheet of scratch paper to acclimate yourself to the format.
- It is important to keep in mind that *except for the data analysis section* all figures on the GRE are *not drawn to scale* (unless otherwise noted).
    - This means all problems with figures are solvable by standard logic and simple mathematical methods - they do not require estimations based on the diagrams. It can also be helpful to draw the two most extreme figures consistent with the given information to clearly delineate all possibilities.
    - You can assume that: (1) straight lines are indeed straight; (2) positions of points, angles, regions, etc. are in the order shown; (3) angle measurements are positive; (4) a circle is a circle, a triangle is a triangle, a quadrilateral is a quadrilateral, etc.; (5) figures lie in a plane unless otherwise specified.
    - In contrast, *data analysis problems* often do require you make estimations based on the diagrams.
- The GRE only deals with real numbers. This will have limited impact but is important in a couple of respects (for instance, square roots of negative numbers are not part of the GRE as they are imaginary).
- Answers on the GRE will always be in simplified form. This means you *must* reduce fractions, reduce roots, etc.
- There are two types of questions that you will see on the quantitative section: Quantitative Comparison (QC) questions and Problem Solving (PS) questions. QC questions and PS questions will be mixed together throughout the quantitative section and can broadly be grouped into four types: arithmetic, algebra, geometry, and trigonometry (data analysis only uses PS questions). However, often questions require knowledge from multiple domains; for example, you will use your knowledge of geometry to set up algebraic equations to solve for an unknown angle or missing side.

The Tao of the GRE™: Quantitative. The key to mastering the quantitative section of the GRE (and the verbal to some extent) is learning how to understand what each question is telling you. The most difficult part of any quantitative problem involves figuring out what information is relevant to solving the problem and deciding which formula(s) and/or technique(s) to use. Once you know these things, the rest is easy (since the math is basic). Learning to let the question unravel and itself and reveal its underlying logic is paramount.

After you learn the facts and formulas tested on the GRE, you'll be able to spot clues that tell you what needs to be done to solve the problem. For example, if you see a perfect square used in a problem (e.g. 4, 9, 16, 25, etc.), you know you will be either squaring or taking a square root of something; similarly, if you must do a difficult calculation that will take longer than 10 seconds (e.g. 4.96 X 99.34), you know you'll need to use estimation or some other method that eliminates the need to do brute calculation.

We will be focusing on learning to listen to what the questions are saying throughout the course.

Use the information in the question to your advantage by letting it tell you how to answer the question. By the end of this course you should have a fully developed mathematical tool kit. Any feature of a question that corresponds to one of your tools (for example, numbers that correspond to special triangle ratios) is a hint that you will need that specific tool to solve the problem.

Affordances and memorization. In order to be able to hear what questions are saying you need to speak their language. Like learning a foreign language, you will want to memorize a number of important and recurrent numbers and relationships: they are the vocabulary of the GRE quantitative section. For example, we recommend memorizing all perfect squares from 1 through 15 as well as 20 (4, 9, 16, 25, etc.) because you will often come across these numbers in the problems. Then, if you encounter a perfect square in a question, you can expect to take a square or square root somewhere along the line. You can think of them as hints.

These hints are an example of what psychologists call *an affordance* (defined by Wikipedia as "a quality of an object, or an environment, that allows an individual to perform an action"). Here affordances refer to pieces of information in the question that tell you what to do. If you memorize all the pieces, then you will be prepared to translate questions into solutions, and efficiently see the quickest path through the problem. If you don't know the math cold, you won't understand what the question is telling you, and you will struggle to figure out which tool to employ when.

4. Quantitative Question Types

***Problem Solving (PS)*.** This type of question tests your ability to decipher and use basic mathematical concepts. They are standard multiple-choice questions with *five* answer choices, and they always have exactly one answer.

Examples:

1. $10^2 =$
   A. 20
   B. 50
   C. 100
   D. 110
   E. 1000

Answer: **C**

2. If $X + Y = Z$, then $Z - Y =$
   A. X
   B. 2X
   C. X + Z
   D. Y - Z
   E. X – Y

Answer: **A**

***Quantitative Comparison (QC)*.** This type of question asks you to compare a quantity in column A with a quantity in column B. These questions often do not have one numerical answer, as you will see once you learn how to approach these question types.

The answer choices for quantitative comparison questions are *always*:

A—If the quantity of column A is greater;
B—If the quantity on column B is greater;
C—If the two quantities (A and B) are EQUAL;
D—If the relationship between the quantities *cannot* be determined.

NOTE: There is NO answer choice E for this type of question.

Examples:

1. <u>Column A</u> <u>Column B</u>

Column A	Column B
15	7

Answer: **A**

2.

Column A	Column B
7	15

Answer: **B**

3.

Column A	Column B
15	15

Answer: **C**

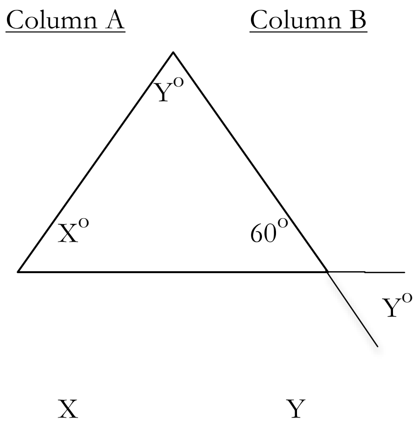
4.

Column A	Column B
X	Y

Answer: **D**

QC Common Information. Often times QC questions refer to a common diagram or chart. When this is the case, the common information will appear centered *above* the column choices.

Example:



Answer: **C**

The triangle above is the common information, and it tells you what you need to know about the answer choices, X and Y, in order to answer the question. In this case, because Y is a vertical angle with the 60° angle, it must also be 60°, and because the other two angles other than X are 60°, then X must also be 60° (since all angles in a triangle must add up to 180°). Therefore the answer is C because X and Y *must be equal.* Notice that if you just eyeballed the triangle, it would look as though X is bigger than Y because the triangle drawn is isosceles: the triangle is indeed equilateral, even though it is not drawn as such.

5. Approaches for Solving Quantitative Comparison (QC) Questions

The Two General Methods. There are two general methods to solving QC problems. Which method you use depends on the nature of the problem.

1. If the problem only has numbers, no variables, *there must be an answer*. This means you can automatically eliminate *D*. Your goal is then to figure out which of the remaining 3 answer choices is correct. Because the answer must be "Column A", "Column B", or "Equal," once you get one answer you are done.
2. When solving QC questions with variables, *D* is not only a possible answer, but also a *likely* answer. The approach you should take is to try to confirm *D* is the answer. If you cannot confirm *D* (e.g. you see a *divergent pattern* or *The Four Test Number Categories* leave you with only one answer), then you can assume the one answer you keep finding is the correct answer. However, you always want to step back and take a few seconds to try and figure out *why* that answer (either A, B, or C) consistently occurs: if you can see a deeper structure or relationship (e.g. *convergence or divergence*) you can *be 100% sure* you have found the right answer. Also, *as a last resort*, if you ever *must* guess, always guess *D*.

Searching For D. In order to confirm that the answer is *D* you need to find two scenarios with conflicting answers. For example if you had to compare the two quantities "X" and "2," and you are given the common information that $X^2 = 4$, then you would try to get 2 scenarios with conflicting answers. You could try X = 2, which would work mathematically and yield an answer of *C*; however, you could then try X = -2, which also works with the common information and yields the answer of *B*. As soon as you find two conflicting answers your work is done: the answer is *D*.

Knowing conflicting answers is a distinct possibility can be unnerving; you might hear that voice in your head asking, "What if there's a secondary answer I just haven't tried yet?" Because there are an infinite number of possibilities to plug in, you can never know for certain if there might be one out there that you didn't test, but don't worry. When plugging in test numbers that follow a pattern (e.g. 2 then 3 then 4), observe how these numbers affect the *behavior* of the quantity. Specifically, do you see a *divergent pattern* (does the difference between column A and B continuously increase) or a *convergent pattern* (does the difference continuously decrease)? Convergence typically means the answer is *D*, since the quantities will eventually intersect!

The Four Test Number Categories. In addition to the tip about plugging in sequences, there are four very specific types of numbers you'll want to test in order to either arrive at D or have confidence that D is not the answer if you cannot find any conflict:

1. <u>1 and 0</u>. You will want to try these because of their many unique properties (e.g. anything multiplied by 0 is 0, anything raised to the 0 power is 1, anything multiplied by 1 is itself, etc.)
2. <u>Positive and negative numbers</u>. It doesn't so much matter which positive and negative numbers you choose to test, just make sure you test one of each because they often cause opposite effects.
3. <u>Integers and fractions</u>. It says fractions here, but more specifically fractions that are between 0 and 1. Just like above, these two types of numbers will often have different and opposing effects.
4. <u>Odd and even numbers</u>. Typically you will not need to be too concerned with this category unless the question specifically asks about odd or even. There are a few other very rare times when odd and even numbers generate divergent behavior, such as when they act as exponents for negative numbers (a negative number to an even power is positive, but to an odd power is negative).

Often the common information will rule out one or some of these types of numbers (e.g. $X > 0$ would rule out both negative numbers and 0). If the common information rules out basically all of them (e.g. $X > 1$), then it is essentially restricting you to a class of numbers that do not typically contain divergent categories, and you just need to plug a couple of numbers from this class in to observe whether the behavior with different numbers is *divergent* (in which case the answer will be *A* or *B*), or is *convergent* (in which case the answer will be *C* or *D*). So: pick at least one number from each permissible category when you are plugging in different values in your quest for *D*.

The Four Test Number Categories
1. 1/0
2. +/-
3. Integers/Fractions ($0 < x < 1$)
4. Odd/Even (in rare cases)

Other Helpful Hints and Strategies. These strategies are subordinate to the general methods laid out above, and are meant to be used *in conjunction* with the methods above.

1. Because QC questions are asking only for a comparison<u>, exact values are not always necessary to obtain</u>. Much as you don't need to know a couple's exact height to determine who is taller or shorter, many QC questions can be answered with only partial or limited knowledge.
2. When the columns have formulas such as "Z – X," <u>solve for the entire thing at once</u>, not the individual pieces. Solve for "Z – X" as a whole: don't try to solve for "Z" then "X" individually so you can do the subtraction.
3. Make the quantities in the two columns look <u>as similar as possible</u>. If you feel like you are comparing apples and oranges, you're missing something major! Oftentimes you will be presented with quantities that *look very different but are actually quite similar*: making them look the same is the first step.
4. When plugging in numbers, it is a good idea to choose *extreme* values. For example, when asked to compare "X" and "Y," if you plug in *1* and *2* the relationship will be a lot less obvious than if you

plug in *1* and *2,000,000*.

5. If the quantities each have multiple pieces (e.g. "$2.8^2 + 9.3$" and "$3.2^2 + 8.5$") it is often helpful to compare them piece by piece. This compliments Strategy #3 above: it is much easier to first compare the squared terms, then compare the two other terms, and last integrate these two sources of information.

6. Approaches for Problem Solving (PS) Questions

- Scratch paper will be your best friend. Always double check to make sure you've copied the correct information from the computer screen onto your work sheet. In addition: you will be granted extra time to read the sectional directions, but after extensive preparation you won't need to read them! Use that extra time before the test starts to create a series of columns with the 5 answer choices (*A, B, C, D,* and *E*) so you can eliminate answer choices on your scratch paper as needed.
- When you come up with an answer, RE-READ THE QUESTION! Sometimes the question being asked is not the intuitive answer derived from simple calculation: it might require one final step!

7. The Data Analysis (DA) Section

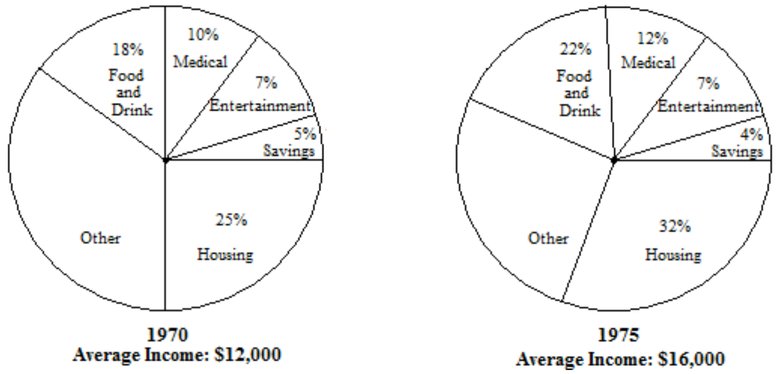
Overview. The DA section will be comprised of only PS question types. This section tests your ability to analyze, interpret, and compare data within a single graph or chart or amongst several graphs or charts. You will see one question at a time with the information next to it on the computer screen.

Spend about one minute orienting yourself with the information presented to ensure you know how to answer the questions. When doing so, check the title of each figure, the scale of each figure (e.g. percentage, 10,000s, etc.), how the information is represented, how the figures relate to each other, and any other relevant information. Just as in the reading section, taking the time to understand the graphs may seem like a waste of precious seconds, but it will actually save untold minutes when attending to the questions, and you'll get them right!

Typically, the calculations needed for this section are minimal. Rather, similar to reading comprehension, an ability to quickly recognize and pull out necessary information is key. Success on the DA section comes through *attention to detail*. Check, double check, and triple check that you are looking at the right graph, reading the right section, and interpreting the information correctly via making necessary scale conversions, understanding units, etc. There are many places for error, so in order to minimize the probability of making a mistake you will want to solve the each question *piecemeal*. Furthermore, it is often helpful to use the side of your scratch paper as an impromptu ruler when comparing different elements of the figures.

Two more things to remember: one, use only the information given in the graph. NEVER infer or "read into" the information presented. Two, occasionally you will be asked for an approximate answer rather than an exact answer. If this happens, round off numbers to make your job as easy as possible.

Example:



For the year in which the average family's housing expenses were $3,000, what were the average family's medical expenses?

A. $600
B. $1,000
C. $1,200
D. $1,920
E. $2,400

Answer: **C**

Example piecemeal method for finding the solution. (1) Find the graph that represents housing expenses of $3000. This requires you analyze and convert both graphs' *percentages* into *numerical amounts* based on average income. This answer tells you to focus on the graph on the left. (2) Now that you know which graph to attend to, find medical expenses. Note they accounted for 10% of total expenditure. (3) Lastly, you must convert that percentage to a numerical value, which yields the answer C because 10% of $12,000 is $1,200.

Notice each step is relatively simple, but such answers typically involve more than one of these simple operations. Rather than attempting one massive, complex, jumbled equation, segment the problem (as above) into a series of simple calculations and observations.

Along the same lines: if you find you need to compare the outputs of two or more calculations, solve each separately, write the answer to each down, then do the comparison as a separate step.

8. Other Helpful Hints

- The GRE tests mathematical abilities most students learn in middle or high school: arithmetic, basic algebra, basic geometry, elementary stats, etc. Even though some of the harder math questions will appear to require more advanced skills, they are always solvable by converting the problem into simpler pieces. There's no reason to rack your brain over college-level calculus or advanced geometry. A directed and well-reasoned review of skills once mastered is all it takes to boost your GRE math score to the levels you desire.
- READ CAREFULLY! Often times ETS disguises simple math problems within long sentences and offers unnecessary information. Learn to decipher word problems and siphon out the necessary information.
- This was previously mentioned, but bears repeating: ALWAYS re-read the question before filling in your answer. ETS often asks the test-taker to do a calculation then seeks an answer that is NOT

derived from said calculation. See the following example: "Tom and Bill can eat 15 cookies in one day. Bill eats 10 cookies in a day. Bill eats cookies for 3 days, and Tom eats cookies for 5 days. *How many cookies would they have left if they started with 100 cookies*?" The italicized text tells you that you must do a final conversion after finishing the main calculations.

- Use the answer choices to guide your work. Often the answer choices can offer clarification as to what ETS is looking for. It is always best to proceed without first looking at the answer choices if possible, but if you are stuck or confused they can be just the ticket to get you started.
- Don't fall for sucker answers! Several answer choices will look juicy at first glance, but do not let this fool you into not doing your work. You need to do the math and think through each and every question—even the easiest ones. This is the only way to avoid the traps. For example, a common QC trap would be to supply the common information that "$X^2=4$" and to put "2" in one column and "X" in the other. Many people quickly choose *C* (they are equal) when the real answer is *D* (cannot be determined) because X could be 2 *or -2*.
- PRACTICE, PRACTICE, PRACTICE! GRE questions are daunting at first and you will often get the wrong answer even when you were sure you nailed it. Take it in stride and before long you will master the test. In addition, when you miss questions during practice tests, come back to the problem and try to figure out how to get the answer, and where you might have gotten off track. This allows you to peak under the hood and examine the logic of the questions so you don't keep making the same mistakes.

ARITHMETIC I

9. Order Of Operations

- When doing arithmetic problems, you must do the operations in a very specific order. If you do not, you're sure to get the wrong answer.
- The order of operations is traditionally captured in the mnemonic "**P**lease **E**xcuse **M**y **D**ear **A**unt **S**ally," with the operations needing to be done in the following order:

  P: Parentheses
  E: Exponents
  M: Multiplication
  D: Division
  A: Addition
  S: Subtraction

10. Arithmetic Properties

- The order in which numbers are *multiplied* or *added* does not change the result (both in terms of where the parentheses are as well as the actual ordering of the numbers):

  (6 + 8) + 3 = 6 + (8 +3) = 3 + 6 + 8 = …
  (6 x 8) x 3 = 6 x (8 x 3) = 3 x 6 x 8 = …

  - The above is *not true* for subtraction or division. Both the actual order of the numbers and the parentheses are important for both division and subtraction, and the analogous relationships with the above examples would not work out!

- Properties of 1
  - The product of any number and 1 is that number: Y x 1 = Y
  - Any number divided by 1 is that number: Y/1 = Y
  - Any number to the power of 1 is that number: $Y^1 = Y$

- Properties of 0
  - The sum of or difference between any number and 0 is that number: Y + 0 = Y; Y – 0 = Y
  - The product of any number and 0 is 0: Y x 0 = 0
  - The quotient of 0 and any number is 0: 0/Y = 0
  - The quotient of any number and 0 is technically undefined, but conceptually is infinity: Y/0 = undefined (as a correct answer on the test); Y/0 = ∞ (as a concept)
  - Any number to the power of 0 is 1: $Y^0 = 1$

11. Number Vocabulary

- Digits
  - There are only 10 digits: 0, 1, 2, 3, 4, 5, 6, 7, 8, 9
  - Every number can be written as a combination of digits
  - The number 67,819.342 has 8 digits: 6, 7, 8, 1, 9, 3, 4, and 2.

    6 is the *ten thousands* digit
    7 is the *thousands* digit
    8 is the *hundreds* digit
    1 is the *tens* digit
    9 is the *units* digit

3 is the *tenths* digit
4 is the *hundredths* digit
2 is the *thousandths* digit

- This number can also be written in expanded notation in 2 ways:
  60,000 + 7,000 + 800 + 10 + 9 + .3 + .04 + .002 and
  6(10,000) + 7(1,000) + 8(100) + 1(10) + 9(1) + 3(1/10) + 4(1/100) + 2(1/1000)

- <u>Natural Numbers</u>
  - The "counting" numbers *not including* zero: 1, 2, 3…

- <u>Whole Numbers</u>
  - The "counting" numbers *including* zero: 0, 1, 2, 3…

- <u>Integers</u>
  - Any real number that can be expressed without using fractions or decimals (although integers can be turned into fractions or decimals, e.g. 3 can also be expressed as 3/1 or 3.00, they *do not require* fractions or decimals for their expression, e.g. 3.5)
  - These are positive and negative whole numbers including zero: -3, -2, -1, 0, 1, 2, 3…
  - The total number of integers is infinite

- <u>Consecutive Numbers</u>
  - Integers listed in increasing order in a parallel and uninterrupted sequence. These need not be increased only by 1. The following are sets of consecutive integers:
    Consecutive integers starting at 0: 0,1,2,3,4,5, …
    Consecutive integers starting at -6: -6, -5, -4, -3, -2, …
    Consecutive odd integers starting at 1: 1, 3, 5, 7, 9, …
    Consecutive prime numbers starting at 2: 2, 3, 5, 7, 11, 13, 17, …
    Consecutive multiples of 3: 3, 6, 9, 12, 15, …

- <u>Rational numbers</u>
  - Any number (including decimals) that can be presented as a fraction. So, for example, π is not a rational number because it cannot be precisely expressed as a fraction (although it is approximately equal to 22/7, or 3.14).

- <u>Irrational Numbers</u>
  - Any number that cannot be written as a fraction, such as π.

- <u>Odd Numbers</u>
  - Numbers not divisible evenly by 2.

- <u>Even Numbers</u>
  - Numbers evenly divisible by 2.
  - Zero is considered an even number.
  - Non-integer fractions are neither even nor odd

<u>Even/Odd Arithmetic Rules</u>
Even + Even = Even
Odd + Odd = Even
Even + Odd = Odd

Even x Even = Even
Odd x Odd = Odd
Even x Odd = Even

- Positive Numbers
  - Numbers greater than zero.
  - These are numbers to the right of zero on the number line.

- Negative Numbers
  - Numbers less than zero.
  - These are numbers to the left of zero on the number line.
  - Thus 0 is the *only number* that is neither negative nor positive.

Positive/Negative Arithmetic Rules
Positive x Positive = Positive
Negative x Negative = Positive
Positive x Negative = Negative
Positive ÷ Positive = Positive
Negative ÷ Negative = Positive
Positive ÷ Negative = Negative

Notice that the positive/negative arithmetic rules involve *multiplication and division*, while the even/odd arithmetic rules involve *addition and multiplication*

- Prime Numbers
  - A number divisible by only the number 1 and itself.
  - Zero and 1 are not prime numbers
  - 2 is the smallest and only even prime number
  - Negative numbers or fractions/decimals cannot be prime.
  - You should memorize the first 20 prime numbers: 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, and 71.

- Composite Numbers
  - Any number divisible by more than just 1 and itself. (Basically any number that is not a prime number.)

- Mixed Numbers
  - A mixed number is a number expressed as an integer and a fraction such as 3 1/4, rather than 13/4. Mixed numbers are the most simplified form of fractions greater than 1, and thus will be the form that answers will take on the test.

- Squares
  - The result when a number is multiplied by itself. 16 is a square because 4 x 4 = 16.
  - The notation for a square is a 2 superscript: so we could write the same equation above as $4^2 = 16$.
  - You will want to memorize the perfect squares of all the numbers from 1-15 and of 20.

- Cubes

- The result when a number is multiplied by itself twice. 64 is a cube because 4 x 4 x 4 = 64.
- The notation for a cube is just like that of a square, but with a 3, so we could write the above equation as $4^3 = 64$.
- You will want to memorize the perfect cubes of all the numbers from 1-5.

- Absolute Values
  - The distance a number is away from zero on a number line. For example, the absolute value of 3 = 3, and the absolute value of -3 = 3.
  - The notation for absolute value is two straight lines around a number, so we could rewrite the above two equations as $|3| = 3$ and $|-3| = 3$.
  - Absolute values are always positive.
  - The absolute value of a positive number is just the number, whereas the absolute value of a negative number is that number without the negative sign
  - It is important to note here that any algebraic equations in absolute value signs always have two solutions (a positive and negative solution), which we will revisit later in this course.

- Remainders
  - The number left over after unevenly dividing one integer into another. For example 27/6 has a remainder of 3 because when dividing 27 by 6 the closest one can get is 24, which is 3 away, so the answer can be expressed as 4 R3 (i.e. 4 with a remainder of 3).
  - A remainder can be used to find a decimal equivalent through fractions, but *is not the decimal equivalent itself!* In our above example we figured that 27/6 = 4 R3, but this is not equivalent to 4.3. To find the decimal equivalent turn the remainder (3) into the numerator of a fraction, with the original divisor (6) as the denominator. Thus, we see that because 4 R3 with 6 as the divisor would be 4 3/6 or 4.5.

- Reciprocals
  - The reciprocal of a number is the number that when multiplied by the first number yields one. For example, the reciprocal of 5 would be 1/5 because 5 X 1/5 = 1.
  - The reciprocal of any number is just that number, expressed as a fraction, and then flipped.
    - The reciprocal of 3 is 1/3 because 3 as a fraction is 3/1, so when flipped, it is 1/3.
    - The reciprocal of 5/4 is 4/5.

- The Number Line
  - The number line is a conceptual metaphor that represents number or quantity as space—specifically as a path through space.
  - In this metaphor, adding is walking to the right, subtracting is walking to the left, and quantity is represented as distance.

12. Symbols

- $=$ Is equal to
- $\neq$ Is not equal to
- $\approx$ Is approximately equal to
- $>$ Is greater than
- $<$ Is less than
- $\geq$ Is greater than or equal to
- $\leq$ Is less than or equal to
- $| \, |$ Absolute value (of whatever is inside lines, e.g. $|x|$)

- || Is parallel to (if variables are outside lines, e.g. x || y)
- ⊥ Is perpendicular to
- % Percentage (this means the decimal has *already been* multiplied by 100)

13. Math Vocabulary

- Sum: the result of addition
  - *Related: total, plus, increase, more than, greater than*
- Difference: the result of subtraction
  - *Related: less, decreased by, reduced by, fewer, have left*
- Product: the result of multiplication
  - *Related: times, at (sometimes)*
- Quotient: the result of division
  - *Related: into, ratio, parts*
  - In the expression a/b=c, c is the quotient
- Divisor: the number you divide by (denominator)
  - In the expression a/b=c, b is the divisor
- Dividend: the number that is being divided (numerator)
  - In the expression a/b=c, a is the dividend
- Numerator: the top number in a fraction
- Denominator: the bottom number in a fraction

14. Factors and Multiples

- Factors
  - A number "X" is a factor of another number "Y" if you can divide "Y" by "X" evenly (e.g. without leaving a remainder).
  - A number's factors are all the numbers it can be evenly *divided by* or *divided into*. There are only a few factors for every number (unlike multiples, which we will cover below, of which there are infinite for every number).
  - Every number has 1 and itself as factors. For a prime number these are its only factors.

Example: What are the factors of 12?

$12 \div 1 = 12$
$12 \div 2 = 6$
$12 \div 3 = 4$
$12 \div 4 = 3$
$12 \div 6 = 2$
$12 \div 12 = 1$

If you'll remember, factors are the numbers a particular number can be evenly divided by or divided into. In the above example, 12 can be divided by 1, 2, 3, 4, 6, and 12, and 12 can be divided into 12, 6, 4, 3, 2, and 1. As you can see, the numbers in both categories are the same. Therefore, the factors of 12 are: *1, 2, 3, 4, 6, and 12*.

To check your answers, multiply corresponding numbers to obtain your original number:

$1 \times 12 = 12$

2 x 6 = 12
3 x 4 = 12
and

4 x 3 = 12
6 x 2 = 12
12 x 1 = 12

Example: What are the factors of 100?

Answer: 1, 2, 4, 5, 10, 20, 25, 50, 100

- Greatest Common Factors
  - A common factor is a factor that two numbers have in common, so the greatest common factor would be the greatest factor that two numbers have in common.
  - All numbers have a common factor of 1, and so you just need to check if they have any greater than 1. If so, you must find the greatest of these common factors.

  Example: What is the greatest common factor of 12 and 18?

  As shown above the factors of 12 are: 1, 2, 3, 4, 6, and 12.
  The factors of 18 are: 1, 2, 3, 6, 9, and 18.
  Thus the common factors of 12 and 18 are: 1, 2, 3, and 6.
  6 is the largest of the common factors, and thus 6 is the *greatest common factor* of 12 and 18.

- Divisibility
  - Factors are related to another important concept: *divisibility*. A number is divisible by another number if the second number is a factor of the first. Or, in other words: a number is divisible by another if it can be divided by that number without leaving a remainder.
  - For example 18 is *divisible* by 3 because 3 is a factor of 18. It is not divisible by 5 because 5 would leave a remainder.
  - If you want to find out if a number, X, is divisible by another number, Y, the best way to do this is to come up with a lower number that you know is divisible by Y, then take the difference between that number and X. If all parts are divisible by Y then X is divisible by Y. This is more easily seen in an example.

Example: Is 936 divisible by 9?
1. First find a number smaller than 936 that is clearly divisible by 9. The most obvious choice here would be 900 (900/9 = 100)
2. Take the difference: 936-900 = 36
3. Then see if the second number is divisible by 9 (if it is, 936 is divisible by 9; if not, it is not): 36/9 = 4. (You will notice that 936/9 = 104, which is the sum of our two divisions.)
4. Iterate this process as needed (see next example)

Example: Is 836 divisible by 6?
1. Find the obvious smaller number: 600 (600/6 = 100)
2. Subtract 836-600 = 236. Find another obvious number: 180 (this is 18 X 100, so 180/6 = 30)
3. Subtract 236 – 180 = 56. At this point you should know if 6 goes into 56 cleanly, which it does not: 56/6 = 9 R2, or 9 2/6 which is 9.3333. Because there is a remainder at the end 836 is not divisible by

6.

4. Notice how once we got our first obvious number out of the way, it was still not obvious whether or not 236 was divisible by 6. So we just repeat the procedure, by choosing 180, which is clearly divisible by 6 since 6 goes into 18 and 180 is just 18 X 10. Then when we get a number less than 100, it should be obvious. If the last one was not obvious to you immediately you need to work on memorizing your times-tables.

- Multiples
  - A multiple of a certain number "X" is X multiplied by any other integer.
  - In other words, a number's multiples are any number it can be *multiplied up to*.
  - Technically, zero is a multiple of every number, but don't expect the GRE to test this fact.

Example: List 5 multiples of the number 10.

Since a multiple is a number multiplied by any integer, pick 5 integers and multiply!

10 x 1 = 10
10 x 2 = 20
10 x 3 = 30
10 x 4 = 40
10 x 5 = 50

So 5 multiples of 10 are: 10, 20, 30, 40, and 50. Are those the only multiples? Of course not—there are infinite multiples for any number.

Negative numbers can also be multiples.

10 x -2 = -20
10 x -5 = -50

So -20 and -50 are also multiples of 10.

- Least Common Multiples
  - A common multiple of two numbers is a number that is a multiple of both. Thus, the least common multiple is the lowest number that is a common multiple.
  - All sets of numbers have a lowest common multiple. Multiplying the two numbers will always yield a common multiple (in the below example 12 x 18 = 216), and so you just need to check and see if there is a lower common multiple than the product of the two numbers.

Example: What is the least common multiple of 12 and 18?

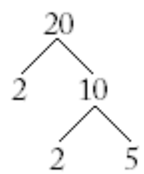
Some low multiples (remember there are an infinite number of multiples) of 12 are: 12, 24, 36, 48, 60, 72 … and some low multiples of 18 are: 18, 36, 54, 72 …
Thus some low common multiples of 12 and 18 are: 36 and 72.
Because 36 is the lowest of these common multiples, it is the *lowest common multiple* of 12 and 18.

15. Prime Factorization

- Every positive integer can be expressed as the product of a group of prime numbers. It is possible (and often times probable) that the same prime will appear more than once in a final factorization.

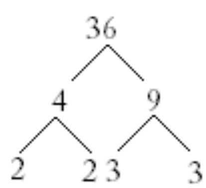
- Prime factorization breaks a number up into its most elementary components. All factors of a number are either prime factors or some combination of prime factors. In fact, you can find all of the factors of a number by doing prime factorization and then grouping the prime factors into all possible combinations.
- The factor tree, and how to do prime factorization:
  1. Take a number and divide it by a factor.
  2. Then write the results of this division on the "branches." If they are both prime numbers, you are done.
  3. If either of them is not a prime number repeat steps 1 and 2. You must keep doing this until all "branches" terminate in prime numbers.
  4. The prime numbers at the end of all your "branches" are the prime factors of your number.

Example: What are the prime factors of 20?



First we divide 20 by 2 and get 2 and 10. 2 is prime so the branch stops there. 10 is not prime so we divide that by 2 and get 2 and 5. Both 2 and 5 are prime numbers so we are done. According to the Factor Tree, the prime factors of 20 are 2, 2, and 5 (2 x 2 x 5 = 20)

Example: What are the prime factors of 36?



First we divide 36 by 4 and get 4 and 9. Neither of these are prime numbers, so we must divide again. We divide 4 by 2, yielding 2 and 2, and divide 9 by 3, yielding 3 and 3. All of these are prime so we stop. According to the factor tree, the prime factors of 36 are 2, 2, 3, and 3 (2 x 2 x 3 x 3 = 36).

ALGEBRA I

16. Algebra and Its Relation to Other Math

Algebra defined. Algebra can be defined as the branch of mathematics concerned with operations and the application of said operations. Operations, in turn, are defined as actions or procedures that can produce new values when given certain input values. You can just think of algebra as the branch of math that deals with the relationships between variables and constants. Any time you need to solve for some unknown quantity, you will use algebra. The algebra on the GRE is very simple, and you probably learned it in high school. In fact, the biggest problem that most students have is that their algebra is rusty! So, if you have algebra anxiety, DON'T FRET! All it takes is a little refresher and a lot of practice.

Algebra and its relation to other math. Algebra will pop up on all kinds of questions and will be integrated with other types of math: many problems will integrate solving for some previously unknown quantity with other concepts. Let algebra and arithmetic provide your foundation for all types of problems.

17. Algebra Concepts and Solving Algebra Problems

Equation Vocabulary

- Equation: a relationship between numbers and/or symbols. Equations express logical relationships, which is crucial to understand in order to solve word problems.
  - An example of what is meant by "formal logical relationship" is the equals sign (=). Many students don't explicitly realize that an equals sign is a logical statement of equivalence. That is, whatever is on one side of the equals sign *is equivalent to* or *the same as* whatever is on the other side.
    - This is why "*whatever you do to one side of an equation you must do to the other*." If you have two things that are equivalent and you change one in one way, then you must also change the other in the same way in order for the equivalence relationship to be maintained. Put more simply, if A = B then A is equivalent to B. If you change A in some way (say by adding 3) and change B in the same way (also adding 3), they are still equivalent.
    - From this simple understanding of the formal logical properties of the equals sign, you can derive the majority of what you need to know for the algebra contained in the GRE.

- Linear Equation: an algebraic equation with only constants and variables to the first power. It is a relationship (=, <, >, ≤, ≥) expressed between values or quantities whose terms are all either constants or variables with an exponent of 1.

- Constant: a term in an algebraic equation that cannot vary. Numbers in equations are always constants because they have a set value. For example, in the equation $3x – 4y = 5$, 3, 4, and 5 are all constants.

- Coefficient: a constant that a variable is multiplied by. In the $3x – 4y = 5$ equation, 3 and 4 are coefficients.

- Exponent: the "power" a term is raised to, written as a superscript number above the term. In the term $5x^2$, for example, the 2 is the exponent.

- Variable: a term in an algebraic equation that can vary and is usually expressed using a letter (but could theoretically be expressed using any arbitrary placeholder). In our 3x – 4y = 5 equation, "x" and "y" are the only variables. Typically in algebra, one is trying to solve for the value of variables that are pre-determined by the equation(s) (i.e. variables whose values are constrained by the equations enough that they can be discovered).

Solving Algebra Equations by Isolating the Variable

- Solving basic algebraic equations: the most basic kind of algebraic equation is linear, with any number of constants and one variable. Even complicated algebraic situations eventually boil down to solving a simple linear equation with a single variable. In fact, all of the more advanced techniques are methods for turning a more complicated situation into this most basic form.

- To solve these equations you must isolate the variable: you must get the variable on one side of the equals sign with all of the constants on the other side. Your work is done when the expression has just the variable by itself (with nothing being done to it).
  1. First, get all instances of the variable on the same side of the equation and get all terms without the variable on the other side. Thus if you have two terms *with* the variable (e.g. 7x – 3 = 3x + 5), then you must get those terms on the same side of the equation (e.g. 7x – 3x – 3 = 5). Conversely, you'll want to get all terms *without* the variable on the other side (e.g. 7x – 3x = 8).
  2. Next, condense all of the terms with the variable into one term (e.g. 4x = 8).
  3. Finally. get anything else attached to the variable to the other side (e.g. x = 8 ÷ 4), and you are done (e.g. x = 2)!

- When isolating a variable, you must follow the rules of algebra. Namely: *whatever you do to one side of the equation, you must also do to the other.*
  1. First, figure out what you want to move (like the 3x and the -3 in the first step above). Then figure out how to get rid of these (if you subtract 3x from 3x, and add 3 to -3, they both would equal 0, and are thus "gone").
  2. Next, do whatever you just did to one side of the equation to the other side. Notice that when 3x is subtracted from the right side above it pops up on the left side of the equation. This is because it had to be subtracted from both sides of the equation.
  3. Repeat this procedure until the variable is isolated on one side, as explained above.

Example: 3x – 4 = 5. Solve this equation in terms of x.

"In terms of…" means "isolate." To isolate x we must get it alone on one side of the equation.

First, let's get rid of that "-4" near the x. And remember: what we do to one side of an equation we must do to the other!

3x – 4 = 5
+4 +4
____
3x = 9

Now, let's get rid of that pesky 3. And remember: what we do to one side of the equation…

3x = 9
÷3 ÷3

$x = 3$

$x = 3$, And we are done!

Now it's your turn:

Example: $5x – 13 = 12 – 20x$. Solve in terms of x.

Answer: $x = 1$

- Literal Equations: equations that contain no numbers, only symbols.
  - Deal with these the same way you would a normal equation, but realize that your answer will be an expression with variables rather than numbers. Learn to get comfortable with treating expressions as though they were variables in their own right.
  - *Applying the Tao of the GRE*™. When the answer or comparison involves an expression, the question is telling you something important, this is a HUGE hint—solve for the expression!

Example: $qp – x = y$. Solve in terms of x.

The first step is figuring out what you are solving for and how to isolate it. Because x is negative, one way to start is to add x to both sides to make it positive before you isolate it.

$qp – x = y$
$+x \quad +x$

$qp = y + x$

Now, let's push that y to the left to isolate the x.

$qp = y + x$
$-y \quad -y$

$qp – y = x$

So: $x = qp – y$!

This is not the only way you could have solved this problem. You could have just as easily isolated -x on the first step by subtracting qp from both sides. Then for the second step you would have had to multiply both sides by -1 (which flips all of the signs, making positive terms negative and vice versa) in order to solve for x.

Example:

Column A	Column B
	$qp – x = y$
$x + y$	qp

As seen in this example, often the questions will ask you about some expression rather than a single variable. If this is the case, solve for the expression as a whole (as though it was an individual term) rather than for each of the variables independently.

In this problem, the first step is to isolate the expression "x + y" on one side of the equation. To do this, add x to both sides.

$$\begin{array}{l} qp - x = y \\ \quad + x \quad + x \\ \hline qp = x + y \end{array}$$

Because qp = x + y, the two expressions are equivalent *no matter what you plug in* for the values of x, y, p, or q. This is precisely what the "=" sign means. Thus:

Answer: **C**

Because this problem is asking you to evaluate and compare expressions (x + y and qp) rather than single terms (x, y, q, or p alone), your best strategy is to solve for the whole expressions *as though they were single terms*. A common mistake many students make is trying to solve for individual terms and then combining (e.g. solve for x, then solve for y, and then add those solutions). DO NOT try to solve for individual terms if you are asked to evaluate an expression—solve for the expression itself!

- Cross Multiplying: a technique for dealing with fractions on opposite sides of an equation. Cross multiplication is done by multiplying the numerator of one fraction with the denominator of the fraction on the opposing side. This is done for both sides.

Example: $b/x = p/q$

Multiply the numerators by the opposing denominators and we get…

$bq = px$

From here solving for any of the variables is simple. You would just divide away the attached coefficient to isolate your variable.

REMEMBER*:* cross multiplication only works with two fractions separated by an equals sign.

- Monomials vs. Polynomials
    - A monomial is an algebraic expression consisting of only one term. $9x$, $4a^2$, and $3mpxz^2$ are all monomials.
    - Do not confuse "one term" with "one variable"! A single term can consist of numerous variables!
    - A polynomial consists of two or more monomial terms. "$x + y$," "$y^2 – x^2$," "$x^2 + 3x + 5y^2$" are all polynomials.

- Working with monomials
    - To add or subtract monomials, deal with the complete term as though it were a single entity

(numeral). You can only add or subtract coefficients of monomials if the monomials have the exact same variable(s) and said variable(s) has the exact same exponents.

- Briefly, $5x + 3x = 8x$, but $5x + 3y$ cannot be further simplified. This is because y is different than x. The first situation is like having 5 apples and then getting 3 more apples; clearly you now have 8 apples. However the second situation is like having 5 apples and then getting 3 oranges; clearly you have 5 apples and 3 oranges, and this cannot be further simplified.
- Likewise, you can add $4x^2 + 3x^2 = 7x^2$, but $4x^2 + 3x$ cannot be further simplified because x and x^2 are different.
- Similarly you can add $5xy + 4xy = 9xy$, but you cannot add $5xy + 4x$. The entire chunk that is not the coefficient must be exactly the same to add or subtract coefficients in monomials.

• Follow the same rules you would with singular digits, remembering to deal with the entire term!

Example:

What is $15x^2yz – 18x^2yz$?

You must deal with the term as a whole. Think of the coefficients as the number of "x^2yz"s that you have. So if you have 15 "x^2yz"s and then you lost 18 "x^2yz"s you would have -3 "x^2yz"s, no matter what "x^2yz"s are exactly:

$15x^2yz$
$-18x^2yz$
____
$-3x^2yz$

- <u>Working with polynomials</u>
  - Treat polynomials as collections of monomials and simply add or subtract all like-terms. One easy way to do this is to arrange like terms in columns, then add or subtract as you would monomials.

Example: What is $(a^2 + ab + b^2) + (4ab -2b^2 + 3a^2)$?

First, we'll align the like terms into columns.

REMEMBER: Don't switch the terms' original values. If it is negative before realignment, it will be negative after realignment! Also, if an entire polynomial is subtracted (e.g. $(a^2 + ab + b^2) – (4ab -2b^2 + 3a^2)$), then all signs must be flipped inside the parentheses (e.g. the above could be expressed as $(a^2 + ab + b^2) + (–4ab + 2b^2 – 3a^2)$).

$a^2 + ab + b^2$
$+3a^2 + 4ab – 2b^2$

Now, add/subtract as if each term were a monomial.

$a^2 + ab + b^2$
$+3a^2 + 4ab – 2b^2$
____
$4a^2 + 5ab – b^2$

This polynomial is now in its most simplified form, since the terms with different variables or variable exponents cannot be combined.

- Working with Absolute Values
  - When solving a linear equation with an absolute value, you must set up two equations. There will be two solutions. Think of this simple example: $|x| = 2$. Here the value of x could be 2 or -2.
  - Likewise, if you had $|x + 3| = 2$, this must be solved for with two equations after you take away the absolute value. Thus you would get:

    $x + 3 = 2$ and $x + 3 = -2$

    $x = -1$ and $x = -5$

    Thus $x = -1, -5$

18. Inequalities

- Inequalities: linear equations in which the relationship between two terms is not one of equivalence but one of disparity (less than, less than or equal to, greater than, greater than or equal to).
- Because the relationship is one of disparity, the solution to a single inequality will always be an infinite set of numbers (e.g. the solution $x > 3$ means x could be any number that is greater than 3 all the way to ∞).
- Instead of using an "=" sign, inequalities use:
  - $>$ (greater than)
  - $<$ (less than)
  - $\geq$ (greater than or equal to)
  - $\leq$ (less than or equal to)
- If the relationship is "less than" or "greater than," the variable could have any value up to the solution, but it *cannot equal the solution*. If your solution is $x > 3$, for example, x could be 4, 100,000,000, or even ∞; x could also be 3.000000000001, but it cannot equal 3 (although it does come infinitely close).
- "Less than or equal to" and "greater than or equal to" function similarly, except that x can actually equal the number in the solution. If $x \geq 3$ then x could be any of the above numbers—but could also actually equal 3.
- When working with inequalities, treat them exactly like normal equations *EXCEPT* for one rule: if you multiply or divide both sides by a negative number, you must flip the sign (i.e. $<$ becomes $>$, and $\geq$ becomes $\leq$)! Only do this for multiplication and division with a negative number.

Example: $-2x + 4 > 6$. Solve in terms of *x*.

Deal with it just as you would a regular linear equation with an equals sign. This means you must isolate your variable:

$$\begin{array}{r} -2x + 4 > 6 \\ -4 \quad -4 \\ \hline -2x > 2 \end{array}$$

Because this next step requires us to divide both sides by a negative number, the sign gets flipped:

$$\frac{-2x}{-2} > \frac{2}{-2}$$

$x < -1$

This means that x can be any value less than -1. Although it could be -1.0000001, it can never be -1.

Now it's your turn:

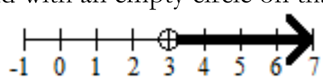
Example: Solve for x: $-7x > 14$

Answer: $x < -2$

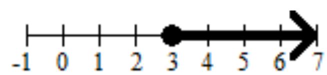
Example: Solve for x: $7x > 14$

Answer: $x > 2$

- Graphing inequalities: because a single inequality allows variables to have a range of values, inequalities are graphed as lines rather than as points (in contrast, relationships with an equals sign are graphed as points since the variable can only have 1 value).
  - Because a "less than" or "greater than" relationship means that the variable can have any value up to *but not including* the number, the line will end with an empty circle on that value.

    The graph of $x > 3$ would look like this: 

  - Because a "less than or equal to" or "greater than or equal to" relationship means that the variable can have any value up to *and including* the number, the line ends with a filled in circle on that value.

    Thus the graph of $x \geq 3$ would look like: 

- Compound inequalities: sometimes you will see questions with a variable that is in several inequality relationships. This will typically be expressed like this: $3 < x < 7$. You should think of this as shorthand for two separate and equally important linear equations: $3 < x$ and $x < 7$.
  - Multiple inequalities mean the value of the variable is constrained by the conditions of both. In the above example, x can have any value between 3 and 7 but cannot equal 3 or 7.
    - There are three possibilities when you have multiple inequalities.
      1. In the above example—in which the variable is greater than the lesser value (3) and less than the greater value (7)—this means the possible values of x lie between these parameters.
      2. If you see the opposite ($3 > x > 7$), this means that x can be any value less than 3 or any value greater than 7, but no values between 3-7.
      3. Sometimes you will see a redundant case ($3 > x < 7$). This means x is less than 7 or less than 3: meaning that clearly it is just less than 7.

19. Algebra: Word Problems

- Word problems require one to transform verbal relationships into math equations, and the key to solving them is acquiring a deep understanding of what *math equations* mean (see the appendix on *The two levels of math that you need to know*). If you find yourself struggling with these problems, it is likely an issue with the conversion from words to math. While there is no magic bullet for solving word problems, there are a few general techniques that you should find helpful:

- When you read a problem, clearly define all the variables given in a list on your scratch paper. This way you know what's known and what's unknown.
- Break the problem into pieces. Don't try to solve the problem in one fell swoop: rather, focus on each piece individually.
- When working on a specific piece, make sure to read the question carefully and think about the relationship between the quantities. Dissect each relationship and create an equation that captures said relationship. Then, briefly analyze your equation, plugging in numbers if you have to, to make sure the equation does indeed accurately capture the relationship from the problem.
- Make sure to generate a separate equation for each relationship that is present in the problem.
- Double check to make sure you have at least as many equations as you do unknown quantities. If you do, solve using algebra (or occasionally other math tools).

20. Algebra: Functions

- GRE function problems are a type of question unique to the GRE that you are unlikely to have seen before (at least in this exact way). They often have weird symbols and require novel operations to be carried out. Don't let this throw you off.
- In a GRE function problem you will see some kind of symbol, such as "#," along with the operation that this symbol defines, such as "x # y = xy + x." All you need to do is carry out the defined operation wherever you see the symbol. So, for the above example, anywhere you see the "#" symbol, multiply the number on the left of it by the number on the right, and then add the number on the left (e.g. 6 # 2 would be 6(2) + 6 = 18). If you think about it, this is exactly what you do anytime you see a math symbol! You just happen to know most symbols and their concurrent calculation. When solving function problems, it helps to just follow the operation like a computer program, and only worry about doing the math once you have everything plugged in properly.
- If you remember algebra well, you might realize this is analogous to functions (thus the name) that you learned with the following notation rather than weird symbols: $f(x) = x - 1$, or $f(x) = x^2 + 3$. Here the only difference is "f(x)" is replaced by some novel symbol.
- These problems take a little practice to get used to. Don't think too hard when you do them; just make sure everything is in the right place as defined by the function.
- Helpful hints:
  - If the symbol appears more than once in the problem, simply solve it piece by piece. If you try to solve the whole thing at once, you greatly increase your chance of making a mistake. For example if the symbol is #, and you see (4 # 5) # 6, first do the (4 # 5); then, only after you've solved 4 # 5 completely should you plug that answer into __ # 6.
  - Sometimes you will see a problem using the same variables to define the function (e.g. "Let a # b = ab + b. Solve for b # a). You can probably see that this can get confusing. A good strategy to avoid confusion is to rewrite the first part with different letters (e.g. "Let x # y = xy + y). This way you don't get mixed up.

Example:

Let x* = x – 1. Solve for 15*/3*.

The first piece of information tells you that when you see anything with an "*" you need to substitute in that value minus 1. First locate all such symbols in the equation, and then just blindly follow the rule:

15* = (15 – 1) = 14
3* - (3 – 1) = 2

Now solve:

14 / 2 = 7
Answer: 7

Let's do another using the same function.

Example:

Let x* = x – 1. Solve (15/3)*

In this case, the symbol * is outside the term (15/3). So, before we can substitute we should solve the parentheses.

15/3 = 5.

Now we have:

5*

Substitute and solve:

5* = (5 – 1) = 4

Answer: 4

Now a more complicated example.

Example:

Let x **Ω** y = xy + 15. Solve (4 + 9) **Ω** (z – 5).

Don't let the greater complexity throw you off. Make sure you know what "x" and "y" correspond to in the new expression, and just plug these terms in like a robot:

In this case "x" corresponds to (4 + 9) because it is on the left of the **Ω**, and "y" corresponds to (z – 5) because it is on the right of the **Ω**, so:
(4 + 9) **Ω** (z – 5) = (4 + 9)(z – 5) + 15

Then just solve:
(4 + 9)(z – 5) + 15
4z – 20 + 9z – 45 + 15
13z – 50

Answer: 13z – 50

APPENDIX: Where Mathematics Comes From and How To Approach Math For the GRE
Disclaimer: This is an essential, but heady overview. Don't stress on this if you don't get it right away! Just take what you can and this will be simplified and explained in class.

The two levels of math that you need to know. All of math is an abstract formal logical structure. As a result, it allows us to carry out logical operations (like adding, subtracting, etc.), by using defined symbols and rules for manipulating those symbols (this is the formal part). To say that math is abstract means that these symbols and rules have no *a priori* content: rather, they can support many different types of content (e.g. math can be used to calculate a grocery bill, solve for the slope of a supply and demand line, figure out a recipe for a cake, etc.): as such, math can be used to model relationships in the world with certain properties (this is how word problems work). You must know (1) *how to use these symbols and rules*, which must be learned through memorization, as well as (2) *what these symbols refer to and what relationships the rules represent*, which must be learned through insight.

Language is an analogous system. Language is also a type of formal logical structure, and it is abstract in the sense that the rules manipulate symbols according to parts of speech and not content. Language has symbols just like math: its symbols are words and letters (or morphemes and phonemes for you sticklers). Language also has rules for manipulating those symbols; after all, you can't just put words together willy-nilly! You need to structure them in a certain way for them to make sense. These rules give utterances meaning; as journalists like to say, "Dog bites man" is not a headline, while "Man bites dog" is news. Math is no different. The rules that govern the operations give math meaning as a whole and allow it to represent everything from the motion of the planets to your grocery bill.

Just as the symbols of language are arbitrary, so are the symbols of math. For example, there is nothing "cat-like" about the sounds "C-A-T," but once the symbol "C-A-T" becomes historically defined as a representation of "the furry pet with four legs that likes to meow," it cannot be easily changed. In exactly the same way, the relationship of equivalence expressed by the "=" symbol could have just as easily been expressed using a "$" symbol, but now that it has been defined it cannot easily be changed. Just as you need to memorize how arbitrary word symbols represent concepts (think of your vocabulary cards), you must memorize how arbitrary math symbols represent concepts.

However, while the rules in math and language are also very similar, the way they are acquired is quite different. Children are born essentially knowing the rules for language, and their understanding of these rules and how they relate to understanding the meaning of language isn't so much *learned* as it is a natural part of development (like having baby teeth replaced by permanent teeth). Similarly, the relationships that the rules of math engender are universal to all human minds. However, in contrast to language, understanding how these rules work and what they do must be learned through explicit instruction. This type of explicit instruction is seriously lacking in American education and is why American students struggle with word problems. Students often don't understand how the symbols and rules map onto concepts and relationships in the world because this is not explicitly taught. So, when we have to decipher a real world event in terms of math (which is what a word problem is supposed to model) we are often dumbfounded as to how to proceed.

How to learn and study math. The above explanations were not just for intellectual gratification (although we hope you found them interesting). They indicate how you should study all math—especially GRE math. You can think of math as having two levels: (1) an arbitrary structure consisting of arbitrary symbols and rules, and (2) a deeper structure consisting of the concepts and relationships that these arbitrary pieces map onto.

Learning (1) without (2) is like learning the vocabulary of a new language—without learning which words are the verbs or how to put sentences together. Learning (2) without (1) is like listening to a foreign language without knowing what any of the words mean. Clearly, both levels must be thoroughly understood if one is to comprehend the true meaning of math and excel on the GRE. However, while these two levels ought to be taught as an integrated whole, they are often taught in a very lopsided manner. There are two major common approaches to teaching mathematics, both of which cover one level well, but leave the other one lacking and underdeveloped.

Military math and hippie math. The first and more common approach can be thought of as "military math." Military math consists of drilling students over and over on mathematical relationships (e.g. times tables), rules (e.g. to divide by a fraction, just multiply by the reciprocal), and rote problem solving (e.g. to find an average add up all the cases and then divide by the number of cases). Because the symbols and rules are arbitrary this is an essential part of an adequate mathematical education, but unfortunately it leaves it up to the student to figure out what it all means—a task often times never realized. The other approach, "hippie math," is also inadequate because often students don't have the symbols and rules down cold. Through "hippie math," students are encouraged to discover the properties and logic of math through exploration, but unfortunately, this often leaves students lacking fluency in how to actually carry out the operations and what the symbols mean. These two approaches both focus on giving students an understanding of one of the two complementary levels of math, but typically students are left with a weak understanding of the other level.

A great mathematical education teaches the concepts that mathematical symbols and rules represent while ensuring fluency in the procedures necessary for correctly manipulating and solving mathematical expressions. As you go through this course, make sure that you know all the arbitrary aspects of math so well you could recite the rules in your sleep (e.g. how to add fractions by finding a common denominator)—but also make sure you know what they mean (e.g. because a fraction is a representation of division, a common denominator is necessary to add numerators). The first aspects must be memorized and are best learned using tools such as drills and flash cards, while the second are best learned through application and reasoning.

As stated earlier, the GRE math section is not a math test but a problem-solving test. If you know all of the symbols and rules but lack the concepts that these symbols and rules represent, you will struggle to solve GRE quantitative problems, which require you to use these mathematical facts to reason your way through. Often students who ace standard high school math tests struggle to answer even simple word problems. If this is you, or if you consider yourself to be "not a math person," then you should pay special attention because this analysis should show you what you were missing, most likely because it was never taught to you.

If you do not know all the math rules and symbols inside and out, your tool-kit will be lacking and you will struggle to "listen to what the questions are telling you." On the other hand, gaps in your conceptual understanding will render you incapable of knowing how to use your tools or even which tool to use. In other words: you need to integrate military and hippie math if you want to master the GRE.

DAY 2
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1. Verbal Section Overview and Question Types

Getting started. The verbal section of the GRE tests not just your vocabulary but also your verbal reasoning skills. While a large vocabulary is essential for success on the test, you must also develop and refine analytical techniques to use this knowledge effectively.

Question Types. There are four types of questions that you will see on the verbal section: Analogies, Antonyms, Sentence Completion (with 1 or 2 blanks), and Reading Comprehension. All of these require an extensive vocabulary, but each must be approached with different techniques. Thus you will be learning and applying two types of tools. The first group of tools is common to all question types because they are designed for the common element in all question types: vocabulary words. The second group of tools will consist of analytical techniques specific to each question type. There is some crossover and interaction between the groups (e.g. using the structure or content of a question may help you understand the definition of an unknown word), but the two are largely independent.

- <u>Learning vocabulary words</u>: having a large vocabulary is the single most important underlying factor for success on the verbal section of the GRE. Even the techniques for questions with unfamiliar words largely rely on your vocabulary (e.g. roots, prefixes/suffixes, etc.). This section will illuminate the best ways for you to study vocabulary.
  - Notecards are the bread and butter of studying vocabulary. There is probably no single better way to memorize vocabulary words than by putting them all on flash cards and studying them until you know every single one. And, they're so small that you can take them with you everywhere: while you are studying for the GRE you should have a stack of vocabulary cards in your pocket or purse at all times, and study them whenever you have a free moment (e.g. on the bus, at the doctor's office, waiting for your boyfriend or girlfriend to show up for dinner . . . you get the point).
  - Studying note cards is pretty straightforward. Just put them in a stack, look at the top card, make your guess, and flip the card. If you get it right you put the card aside; if you get it wrong it goes back to the bottom of the stack. Make sure you actually got the definition right; if you hedge you will only be cheating yourself. Coming close, getting it last time, "knowing it" but not producing it, etc.: none of these count, and the cards should go back to the bottom of the stack. Just keep going through the stack until you have put them all aside.
  - Getting the definition correct once does not mean you've got it down. So, just do this over and over and over and over and… One helpful method is to set up a series of boxes. When you get a card right it goes in the first box. Go through all the cards until they are in the first box. Then pull them out, and when you get them right again they go in the second box. Repeat this process with a minimum of 6 boxes.
  - As part of the course you received a list of vocabulary words, as well as a list of roots, prefixes, and suffixes. Every item on both lists needs to go on a card. When you start studying you should start by <u>memorizing all of the roots first</u>. Memorizing the roots first will later help you learn any words that contain those roots, facilitating later memorization, and will help you constantly reinforce your memory of the root. You will start to see patterns and clusters in the vocabulary words that will both help you learn them and help you if you encounter a similar word that you are unfamiliar with on the test.
  - Once you know all your roots, you need to memorize all of the words on the wordlist you received with your course materials. This list has been specifically tailored to the GRE, which tends to repeat words!
  - If you ever encounter a word you don't know during the class, in the homework, or on a practice test, you should look it up and throw it on a flash card. You won't be able to do this right when

you encounter the word, so you should always bring a highlighter to class and have one handy when practicing at home.

- The best way to learn any language, whether native or foreign to you, is through full immersion. In addition to rote memorization, it is important that you try to use your new words in a natural setting. You will be memorizing A LOT of vocabulary words for the GRE, which means they will easily slip away if you don't begin to use them. Trying to work them into your verbal repertoire will keep you in a lexical mindset, helping you not only maintain recent additions, but also notice and pick up even more.
- Finally, the most natural way to learn vocabulary is by reading. You should make a point to read well-written publications that you enjoy: those like *The New York Times*, *The Economist*, and *The New Yorker* will be full of words you should know for the test. As you read, pay attention to how familiar words are used in context, and throw all unfamiliar words onto a note card. You will start to notice your new words popping up all over the place. Pay special attention so that you can keep reinforcing your memories!

2. General Vocabulary Techniques

Paradoxical Threshold Calibration. Somewhat paradoxically, the more you know about a question the more reluctant you should be to throw out answers, and the less you know the more easily you should throw out answers. When you know the words well you should have a high threshold for throwing answer choices out, and only throw them out when you are positive they are wrong. If you don't know the definitions, you should have a very low threshold for throwing them out, and act decisively to throw out answer choices quickly and without regret.

- If you are doing a problem in which you know all the definitions perfectly for every word, then your threshold for throwing out wrong answers should be very high. Don't be too trigger-happy in throwing out unappealing answer choices: don't throw out answers easily! Trust that with this full information and with your best tools, the elimination process will take care of itself.
- Conversely, when you are lacking the information you need—if you don't know the words, for example—you should have an extremely low threshold for discarding answer choices. If you find any reason to throw an answer choice out (e.g. it looks like a sucker answer, it seems unlikely that it would be an answer, it looked at you funny, whatever) trust your instincts, and do it decisively. DON'T SECOND-GUESS YOURSELF! If you don't know the words in the first place, that means you have nothing to lose. Don't stall out, apply the methods you will learn here, go to town eliminating, make your best guess, and move on!
- For all situations in between knowing everything and knowing nothing your threshold value should vary accordingly.

Precise Techniques. Use these techniques when you know the definition of a word well.

- The best approach for any situation that demands a definition for a word is to generate a clear, concrete precise definition that is as complete as possible. If you know the definition of a word, do not settle for a vague "sense" of the word. The more precisely you can define a word, the better position you are in. This means that when you make flashcards and study them you must not take any shortcuts: write the definitions down and work on memorizing them verbatim. Pay attention to all of the definitions' features and components, including qualifications that differentiate a word from other similar words and connotations that "color" a word.
- Many words on the GRE have multiple definitions, and the GRE uses less common definitions all the time (especially if they are a different part of speech than the most common definition). If a word has a number of distinct definitions you should memorize each, especially if they correspond to different parts of speech, unless they are all very similar (in which case, select the best one). When

you get a word with multiple definitions on a question, don't list all of them immediately, but clearly define 1 or 2 in a clear way as described above. If neither of these definitions seems to work, or if the answer choices are a different part of speech then the definition you are using, start again with a different definition. If none of these definitions work, it is always possible that the question is using an obscure definition that you do not know, in which case you should revert to your approximate techniques.

Approximate Techniques. Use these for words whose definition you do not know well, or you are unfamiliar with. They range from moderately powerful to relatively weak (but better than nothing), depending on the information you have. They can be used independently or in conjunction with one another, and whenever possible should be used in conjunction with the last strategy listed below (using clues from the question to help).

- When you do not know the definition of a word you should bring all of your knowledge to bear on the issue. Anything that you can use to increase what you know about a word will increase the probability of honing in on the correct answer and help you eliminate wrong answers.
- The first thing you should do if you do not know a word is analyze prefixes and suffixes to determine what part of speech it is and glean anything else you can (e.g. if you see "–ist" as a suffix, this would mean it is not only a noun, but also some kind of person).
- Additionally, analyze the roots. You will have a number of roots memorized from the list, but you may also encounter some you don't know.
  - If you see a root you don't know the exact meaning of, think of other words you know that have that root. Try to generate at least 3, and then "average" these words so you can pull out what they all have in common.
- Break the word up into syllables. Roots (and prefixes/suffixes) tend to occur as syllabic units. Some roots are two syllables, but most are only one, and roots will almost always break at the syllables.
- Because of the way English has evolved historically there are two general classes of words from two different sources. Words with 2 syllables or less are generally "feeling words" and are derived from Anglo Saxon. Words with 3 or more syllables are generally derived from Latin or Greek and are more "intellectual words." This has 2 implications: (1) if the word has more than 2 syllables, look for Greek and Latin roots, and (2) sound structure (we'll talk about this more in a bit) tends to be a better clue to meaning for words with less than 2 syllables.
- If you know a foreign language, especially a Romance language (Italian, Spanish, French, or Portuguese), you can often use cognates (words that are similar in the two languages) to figure out a word you don't know. The Romance languages are especially helpful for words with 3 or more syllables, or parts of these words.
- Think about where you might have heard the word or similar words before. As with unfamiliar roots, try to think of a few contexts in which you have heard the word, or a few similar words and then "average" across them to distill the common feature(s).
- Gauge the temperature of the word. You might not know anything about a word other than whether it is positive or negative. This can be sufficient for getting the right answer. At the very least, it can help you narrow down the choices.
- Even if you come across a word you have never seen before and you can't get anywhere with the above methods, you can still often get some information from sound structure. There are universal features of the human cognition of sound that often "leak" into words such that you can get at least a little bit of information just from the way a word sounds. If you have nothing else to go on, just go with your first impression of the word and trust your intuition. It sounds crazy, but it often works; however, you really have to trust your gut when you do this. If you don't know the word anyways, what do you have to lose?
- Look for consistency in parts of speech. The stimulus and any potentially correct answer choice will

always be the same part of speech. (Note that this does not mean that the two stimulus words in an analogy question must always be the same part of speech; often they are not. However if you have an analogy with *adjective : noun*, then you know all the choices must also be *adjective : noun*, and vice versa.)

- Sometimes you can use clues from the question to get some sense of what a word means. We have placed this technique at the end of this section because it is a bit of an anomaly: it is the only general vocabulary technique that must be used in conjunction with the question-specific techniques (given below in each question-type section), which can all be used independently. Rather than being a method for discovering the meaning of a word, this technique is more like an amplifier, greatly increasing the power of the above strategy or strategies it is used with! This technique also serves as a bridge between all of these general techniques—both precise and approximate—and question-specific techniques.

3. General Verbal Strategies

The verbal GRE requires analytical thinking. Language can be highly subjective, ambiguous, and open to interpretation. If you have ever explicated a poem to uncover different levels of meaning, completely misunderstood something despite hearing it clearly, or had a very different interpretation of a statement than the person right next to you, then you are familiar with these aspects of language. In many domains (poetry, word play, jokes, etc.), this subjectivity is the most important aspect of language.

Language also has many precise and objective properties. Because the verbal questions must all have right and wrong answers, they must be based upon those features of language that are precise and concrete enough to support objectively correct and incorrect answers. This does not mean that elements such as connotation are not important (quite the contrary, connotation is very important), but it does mean that you need to focus on clarity and precision from definitions to analysis, and try to get away from the subjectivity that generally pervades our use of language.

It requires some work to shift one's linguistic view away from the imprecise and fuzzy day-to-day use of language to clearly see the crisp and concrete features that the GRE questions are based on. The precise and objective use of language is often unfamiliar; it takes practice to master and deeper analysis to understand.

Often when people begin working on the verbal GRE problems, they feel that the problems are very subjective and open to interpretation. You might pick what you are certain is the correct answer only to find that the right answer was instead an option that you had confidently ruled out. If you experience this, dissect the definitions and relationships in the question to understand the concrete features defining the right and wrong (but perhaps tempting) answers. When doing this, the questions will eventually lose their arbitrary appearance as their objective features come into relief. Only through repeated exposure to the verbal GRE and careful analysis of seemingly arbitrary questions will you see the concrete linguistic features necessary for confidently differentiating right and wrong answers.

We don't even realize how sloppy everyday language is because it need not be precise or concrete for most communicative purposes. For example, often when reading we glaze over unfamiliar words without even realizing we have done so; we use words without knowing precisely what they mean, and likely our imprecise understanding works just fine; we use words whose meaning we don't know at all because they are contained in opaque clichés and expressions (e.g. what are you actually saying when you excuse something due to "extenuating" circumstances?).

One reason we don't notice the imprecise and fuzzy nature of our everyday language is because we are accustomed to "filling in the blanks" both consciously and unconsciously. It is important to be aware of this process and avoid it to prevent your biases from affecting your interpretations. Many features of our analytical techniques are specifically designed to help you do this: most of them involve trying to make a prediction about what the answer will be before looking at the answer choices.

Once you have read the answer choices, they are in your head. They can often push around your thoughts in ways that you are not aware of, which often leads to misinterpretation and consequently the wrong answer. To prevent this, try not to read the answer choices until you already have made a prediction about what you think the answer might be.

Biases and unconscious "filling in" processes can negatively affect all types of questions, but they are especially damaging in the reading comprehension. When we read, we all try to make sense of what is being said by bringing in other knowledge, and, depending on what we are reading, we typically are either trying to help the author get his or her point across to us or attempting to pick the piece apart and poke holes wherever possible. However, the GRE reading comprehension specifically asks you questions that must be answered only from what is actually in the reading passage (or can be clearly inferred or predicted from it without other outside information). Our analytical techniques for the reading comprehension are also constructed to try and eliminate any additions into the passage that aren't already there (much like our techniques for antonyms, analogies, and sentence completions).

The Tao of the GRE™: Verbal. Excelling on the verbal section of the GRE requires an approach that immediately reveals the optimal analytical technique you should employ, allowing you to answer any problem quickly and confidently. By breaking every problem into clear and simple steps, each technique eliminates subjectivity and maximizes the probability of success. On the quantitative section, the *Tao*-approach is based on letting the question guide your choice of tool from your mathematical tool belt. The *Tao*-approach on the verbal is different; rather than choose tools from a box, you will swiftly run through a hierarchical checklist of techniques specific to each question that will immediately provide you with the optimal analytical technique for the situation given what you know about the words you are being tested on.

You want to master the few simple analytical techniques and become comfortable with the language of the GRE so that on every question you can hit the ground running and cruise through the test with confidence. You will inevitably come across words that you do not know, and you will inevitably get questions wrong. But this should not bother you: worrying about it will only have a negative impact on your score. Using the *Tao*-approach, your goal should not be to get the right answer every time, but rather to give yourself the highest probability of getting the right answer each time. This subtle shift in thinking will increase your score by freeing you to make decisive choices, knowing that regardless of whether you get any individual question right or wrong, you have used the approach with the highest probability of success. It will help you to not second-guess yourself (which is a major source of error on verbal questions) and lead you to the best possible overall score.

THE TAO OF THE GRE™: VERBAL

At the start of each problem, run down the analytical checklist to find the best technique available. Then immediately employ this technique in a precise and disciplined manner. Your goal is not to make sure you get every single question right: it's to use the method that gives you the highest probability of success on any single question. Move through questions decisively and confidently, knowing that the analytical technique indicated by the checklist has given you the highest probability of success!

- The analytical checklists are both hierarchies and road maps.
- Integrate the specific question-based checklists with your general vocabulary tools.
- All analytical checklists for each question type are outlined in their respective sections.

4. Analogies

The Question. GRE analogy questions test not only your vocabulary, but also your ability to determine the relationship between a set of words. Once the relationship has been determined, it's your job to select the answer choice that contains the same relationship in the same order.

- GRE analogy questions will always look like the below example. There will be two words in capital letters separated by a colon. These are words are called the stimulus and provide the relationship that you will be comparing the answer choices to.
  - The direction of the relationship is very important. The **answer must have the same relationship in the same direction as the stimulus**. The relationship for the below example is "a jug is a type of container," and there are two answer choices with the same relationship "a crocodile is a type of reptile" and a "fork is a type of utensil." However "fork : utensil" is wrong because the relationship goes from the 1st word to the 2nd word, whereas the relationship in the stimulus is the 2nd word to the 1st word.

CONTAINER : JUG ::
(A) book : library
(B) reptile : crocodile
(C) house : mansion
(D) fork : utensil
(E) cloud : sky

The Basics. There will always be a relationship in the analogy. This is not a Rorschach inkblot test—the GRE will never present an open-ended question.

- Connector Sentences (a.k.a. relational sentences, or bridges)
  - Connector sentences are sentences you create that clearly state the relationship in the stimulus. The stimulus words allow you to create the sentence. Then you can pull those stimulus words out, yielding a precise scaffold, which you can use to evaluate the answer choices. It is essential that you use the exact same sentence in the same direction (this is the disciplined analytical part that safeguards against biases).
  - The worst approach to an analogy problem would be to use a vague concept, feeling, or intuition. Use a concrete connector sentence and apply this to all of the answer choices in a systematic way.

Example:
KITTEN : CAT
(A) dog : mouse
(B) fish : whale
(C) kid : goat
(D) sheep : ewe
(E) goose : gosling

For the connector sentence start with the obvious relationship: "A kitten is a baby cat."

Now, pull out the stimulus words so we can use the exact same sentence to test all of our answer choices: "A ________ is a baby ______"

Now plug them all in and see what works.

A dog is a baby mouse → NO
A fish is a baby whale → NO
A kid is a baby goat → YES
A sheep is a baby ewe → NO
A goose is a baby gosling → NO (a gosling is a baby goose, but the order is reversed)

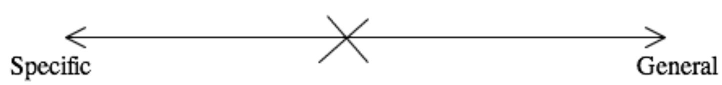
Answer: (C)

- Building Strong Connector Sentences: a good connector sentence will explicitly capture a *concrete* relationship between the stimulus words. Sometimes, your first connector sentence will either work for no answers or for more than one answer. The connector sentence method below deals with this problem by first testing all answer choices, and then looping back to the beginning to alter or recreate the connector sentence until it isolates just one answer.

Let's see how the process works with the following example.

MANSION : HOUSE ::
(A) car : truck
(B) boulder : rock
(C) cookie : cracker
(D) cedar : tree
(E) sun : moon

To build your first connector sentence, use the most obvious relationship between the two stimulus words. This should be a relationship between the two words (e.g. "A mansion is a type of house"); it should NOT ever be both words' relationship to some other entity (e.g. "Both mansions and houses are buildings"). Think of the various kinds of possible relationships between two words as if they were sitting on a continuum that runs from specific to general. Your first sentence should be right in the middle of this spectrum (see below); this is called the *natural level*. If the first try doesn't work, the method is self-correcting, so don't spend too much time trying to perfect your sentence on the first try.



Once you have specified a clear and explicit relationship between the stimulus words you need to pull out the stimulus words (e.g. "A _________ is a type of __________"). It is essential that you do not change the sentence at all except for this extraction.

Then apply your sentence to all answer choices: go down the list and plug each set of words into the blanks. Keep any answers for which the relationship holds, and throw out all others. As a general rule, you should be decisive and harsh when throwing out answer choices. The only possibilities here are "yes, these words work" and "no, they don't": if you find yourself saying, "Well, maybe…" throw the choice out.

(A) A car is a type of truck → NO
(B) A boulder is a type of rock → YES

(C) A cookie is a type of cracker → NO
(D) A cedar is a type of tree → YES
(E) A sun is a type of moon → NO

If you end up with one and only one answer choice at this point you are done. However, if you end up with more than one, your sentence was probably too general: you need to go back and make it more specific. If you end up with no answers left, either your sentence was too specific or you have the wrong relationship: either way you must go back and change the sentence.

In our example we have two answers remaining, which means our sentence was too general, and we have to move to the left on the continuum and make the sentence more specific.

Let's try "A mansion is a large type of house." Now we must go back through and reapply this to all of our answers:
(A) A car is a large type of truck → NO
(B) A boulder is a large type of rock → YES
(C) A cookie is a large type of cracker → NO
(D) A cedar is a large type of tree → NO
(E) A sun is a large type of moon → NO

As soon as you are left with just one answer as we have above, you are done. This is what we mean when we say that this method is self-correcting.

- Technique 1: Connector Sentences
  - *Step 1:* Define both words as clearly as possible and consider their relationship.
  - *Step 2:* Construct a connector sentence relating the two stimulus words to each other.
  - *Step 3:* Pull out the stimulus words and plug the answer choices into the connector sentence in the slots where the stimulus words were. Keep answers that work, throw out ones that do not. Remember: (1) don't change the connector sentence at all, and (2) order matters.
  - *Step 4:* If you end up with just one answer, you're done. If you end up with more than one, your sentence was too general, and if you end up with no answers, your sentence was either too specific or wrong: in both cases, go back to Step 2 and modify as needed. Keep looping through until you are left with just one answer.

- Technique 2: Formulaic Methods. Some types of analogies are squirrely when you try to force them into a connector sentence. If this happens, these formulaic methods are almost as strong (and often quicker) as the connector sentence method. There are a couple of general types.
  - The Formulas: some relationships can be captured in mathematical notation, which is especially great because these can be some of the hardest ones to get into connector sentences. The formulas are almost as powerful as connector sentences, except that they can only represent discrete relationships.
    - $X : X^2$ This formula captures a relationship that differs by degree. The example given above of MANSION : HOUSE would be $X^2 : X$. The X^2 represents the thing that differs from the natural level. It is important to note that just as squares can be larger (positive integers > 1) or smaller (fractions), so can this relationship. For example, both PEBBLE : ROCK and BOULDER : ROCK fall in this category.
    - $X : -X$ This formula captures a relationship that differs by valence (or

connotation). It represents words that mean the same thing except one is positive and one is negative, such as FRUGAL : STINGY.

- $X : -X^2$ This formula integrates the last two. It expresses a relationship that differs both in degree as well as valence. Continuing with the example from the last one, a good example of this would be FRUGAL : MISERLY.
- $X : X^3$ This formula bridges the two possible directions of the $X : X^2$ relationship: the X is significantly smaller than the X^3. To illustrate both directions of the $X : X^2$ relationship, we gave the examples of BOULDER : ROCK and PEBBLE : ROCK. $X : X^3$ would then be PEBBLE : BOULDER. Another example would be SHACK : MANSION.
- Synonyms and Antonyms: because you don't want to build connector sentences that reference a third element, synonyms and antonyms can be tough to wrangle into a reasonable connector sentence. Thus, this is a powerful technique that does not involve a connector sentence. Just state it: "frugal and thrifty are synonyms."
- Approximate Synonyms and Approximate Antonyms: sometimes you will see questions with approximate synonyms or antonyms that are difficult to deal with. These are stimulus words that are basically synonyms or antonyms, but they are different parts of speech, so they aren't synonyms or antonyms proper. Often these can be put in connector sentences—if so, do so (e.g. for DEVELOP : STAGNANT, "something that does not develop is stagnant")—but if they can't or the sentence eludes you, you can just state it (e.g. "develop and stagnant are approximate antonyms").

- Technique 3: For when you only know 1 of the 2 stimulus words
  - Use your approximate vocabulary techniques to get as much information about the other word as possible.
  - Try to see if the question is intentionally set up as a problem-solving task in which you are not expected to know one of the words.
  - For example, there exists a question with following stimulus: COINS : NUMISMATIST. Do the GRE folks really expect people to know what a numismatist is? Probably not! And the other word ("coins") is oddly simple and common next to this enigma. Let's do a quick problem-solving analysis. The strange word has the suffix "-ist," which means it must be a person. Now, seriously: how many relationships can a person have to coins? They could mint them; they could collect them; they could fear them maybe… well, it's not the last one because it does not have "phobia" in it. So, we're thinking a "numismatist" is either a minter or collector. In this question, the answer choices make it clear a numismatist is indeed a collector of coins. What's the take-home message here? DON'T PANIC if you see a crazy word. They don't expect you to have the *OED* memorized: they expect you to be able to solve problems creatively and intelligently.
  - If your analysis of the stimulus words does not help you reason through the relationship, try and get a rough assessment of what the mystery word could be, make your best guess at a good connector sentence, then move to the answer choices and see if your guess works.
  - Eliminate those choices without a clear relationship. There are problem-solving questions in which you don't need to know the stimulus words: you just need to know that some of the answer choices are unrelated pairs. Clearly a pair of words that are unrelated could never be an answer in an analogy question! In addition, pairs without relationships are often set up as sucker answers, so they will be words that commonly occur together, seem related but aren't, etc. For example the following pairs could all be eliminated, always:

    sun : moon
    pitch : black

owl : eagle

- Eliminate any pairs of answers that have the same relationship. While sometimes the correct answer does have a similar relationship to another answer choice, if you don't know the definitions, you will not be able to tease them apart even if one could be the correct answer. These are unlikely choices when the stimulus is difficult. (Think back to what we discussed about lowering your elimination threshold for uncertain questions!)
- Eliminate those that look like sucker answers even if they do have a relationship. If you are uncertain on the question, lowering your threshold for elimination means throwing out answers that might be possible, but still seem unlikely.
- Work backwards with the remaining answers: make connector sentences for each answer choice and plug the stimulus words into each connector sentence to see if that answer choice seems plausible.
- Try to determine the sentence that best fits the word you know, and its possible relationships to the mystery word.

- <u>Technique 4: For when you don't know either stimulus word</u>
  - Use your approximate vocabulary techniques to get as much information about both words as possible (as in Technique 3).
  - Analyze it to see if it might be set up as a problem-solving task (as in Technique 3). This goes hand in hand with using your vocabulary tools to get your best guess, no matter how vague: even if you just think one sounds positive and the other sounds negative, that is better than nothing. Maybe that was exactly the problem-solving task that the question was designed for: checking if you could get a toe-hold (an uncertain inference) off of nothing more than sound structure or the presence of functional prefixes or suffixes!
  - Work backwards (as in Technique 3).
  - Eliminate answer choices without a clear relationship (as in Technique 3).
  - Eliminate answer choices that have the same relationship (as in Technique 3).
  - Eliminate those that look like sucker answers even if they do have a relationship (as in Technique 3).
  - Make an educated guess about which remaining answer pair has the strongest relationship and pick that one. Just remember, you have nothing to lose if you don't know the words so be decisive and confident and move on!

- <u>Common Sucker Answers</u>: there are some common types of tempting foils. Being aware of this will help raise a red flag whenever you see such answer choices. This does not mean that in all cases these can never be the answer (although some like *the unrelated* cannot), but rather that if you suspect that the answer does have one of the following properties, you should triple-check that it is in fact the answer and you are not simply being lured in. Furthermore, if you are in a situation where you don't know the stimulus words, these answer types should all be immediately eliminated. Remember to be decisive and relentless when you do not know all the definitions.
  - *The Unrelated*: if the two answer choice words do not have a clear and concrete relationship, they can be eliminated. The answer will never be unrelated words. Many times they will try to make unrelated words tempting by giving them some of the below properties, like oral familiarity.
  - *The Too Related*: a favorite trick of the GRE is to offer an answer choice with words from the same domain as the question words. For example, if the stimulus contains a tool (hammer : carpenter) an answer choice will do the same (screw-driver : painter). Similar fields don't always signal a wrong answer, but often they do signal easily dismantled traps.
  - *The Orally Familiar*: these are answer choices that are commonly heard or spoken together,

but have no true relationship (e.g. unforeseen : circumstances or pitch : black)

- *The Reverse Analogy*: remember that order matters! Beware of answer choices that would work if only the words were reversed.
- *The Opposite Analogy*: similarly, certain answer choices will contain the opposite of the connector sentence you've written. For example if you get BOULDER : ROCK in the stimulus the connector sentence would be "A boulder is a large rock." You then might see pebble : rock in the answer choices: although this is $x^2 : x$, the relationship of degree goes in the opposite direction.
- *The Triadic Relationship*: the connector sentence must explicitly capture a concrete relationship between the two stimulus words, not their relationship to some third entity. For example the answer choice gold : silver is tempting because they share a relationship with a third entity (e.g. "Gold and silver are both precious metals"). But they are not concretely related to each other, which means they can never be an answer for any analogy question.

- Common Relationships: become familiar with these. You should always construct your own connector sentence for each question and not rely on a limited set of pre-fabricated connector sentences. Constructing your own connector sentence and using the connector sentence method assures you the flexibility you will need to always find the right sentence. Nevertheless, knowing these common relationships can increase your efficiency in creating the correct connector sentence.

  - *Classifications*
    - *Definitional* (this is the strongest possible type of relationship and should always be used when present, e.g. "Reticent means to be reserved")
      Example: reticent : reserved
    - *General to Specific* (e.g "A cardinal is a type of bird")
      Example: bird : cardinal
    - *Person to Characteristic* (e.g. "A bodybuilder is someone characterized by strength")
      Example: bodybuilder : strength
    - *Synonyms* (e.g. "Reticent and taciturn are synonyms")
      Example: reticent : taciturn
    - *Antonyms* (e.g. "Reticent and effusive are antonyms")
      Example: reticent : effusive
    - *Male to Female or Animal to Sex*, like deer : doe (it can be hard to make a sentence for these, but an approximation can work: e.g. "A buck is a male doe." NOTE: this is just an approximation because it does not include the femaleness of the doe, but keeping that in mind, this will work just fine)
      Example: buck : doe
    - *Adult to Youth* (e.g., a kitten is a baby cat)
      Example: kitten : cat
    - *Virtue to Failing* (this can be expressed as a definitional lack relationship, e.g. "To have cowardice is to lack fortitude")
      Example: fortitude : cowardice
    - *Element to Greater Degree/Element to Lesser Degree* (this is an $X : X^2$ relationship and can be expressed as such, e.g. "Snow and blizzard are an $X : X^2$")
      Example: snow : blizzard

  - *Structurals*
    - *Part to Whole* (e.g. "A battalion is made up of soldiers")

Example: soldier : battalion
    - *Whole to Part* (e.g. "A finger is a part of a hand")
      Example: finger : hand

- *Operationals*
    - *Time to Time* (e.g. "An hour is comprised of minutes")
      Example: minute : hour
    - *Complete Operation to Stage* (e.g. "A game is made up of innings")
      Example: game : inning

- *Overlapping*
    - *User to Tool* (e.g. "A stethoscope is a tool used by a doctor")
      Example: doctor : stethoscope
    - *Creator to Creation* (e.g. "A painter makes portraits")
      Example: painter : portrait
    - *Cause to Effect* (e.g. "A fire creates heat")
      Example: fire : heat
    - *Profession to Cause* (e.g "Education is the job of a professor")
      Example: professor : education
    - *Instrument to Function* (e.g. "A ruler is a tool of measurement")
      Example: ruler : measurement
    - *Symbol to Institution* (e.g. "The bald eagle is a symbol of the U.S.")
      Example: bald eagle : United States of America
    - *Reward to Action* (e.g. "Bravery is rewarded with commendation")
      Example: commendation : bravery
    - *Object to Hindering Obstacle* (e.g. "A flat tire will hinder a bike")
      Example: bike : flat tire
    - *State to Satisfaction* (e.g. "Hunger is a desire for food")
      Example: hunger : food
    - *Object to Medium* (e.g. "A boat operates on water")
      Example: boat : water
    - *Object to Operator* (e.g. "A safe is protected with a combination")
      Example: safe : combination
    - *Object to Material* (e.g. "Jeans are made of denim")
      Example: jeans : denim

ANALOGY ANALYTICAL CHECKLIST

- Technique 1: Connector sentences
- Technique 2: Formulaic methods
- Technique 3: When you only know 1 of the words
- Technique 4: When you don't know either word

5. Antonyms

The Question. Antonym questions present a word and ask you to select the answer choice that defines the OPPOSITE meaning of the word.

- Antonym questions will always have the structure of the below example. The stimulus will be in caps, and there will be a list of answer choices below.

ALTRUISTIC :
(A) narcissistic
(B) poor
(C) rich
(D) selfish
(E) rude

The Basics.

- Secondary meanings
  - Often the stimulus word will have more than one meaning. It's best to start with only one or two of the most obvious meanings and then move on to others if these do not work.
  - If you know the stimulus word well and you can't find any antonyms for the definition you are using, this likely means the question is based on another definition. If you can't find an answer right away, start thinking of other definitions.
  - The definition used for a stimulus word must have an antonym (as must any correct answer choice!). Often words with many definitions will only have a couple that have clear opposites. Keeping this in mind will help you narrow the number of definitions you have to consider.
  - If the GRE tests a secondary definition, there will be no possible answers for the primary definition. It should be clear which definition is being used, and you will never have to decide between antonyms of different definitions.
  - If you are having trouble figuring out which definition the question relies on, look at the answer choices. The stimulus and answers will all be the same part of speech, which may help you know which definition of the stimulus you should focus on.

- Opposite of what?
  - The stimulus word definition can have multiple components, and an antonym could be the opposite of any of these components. This means that a single stimulus can have multiple distinct antonym possibilities.
    - For example, the word "plummet" means "to decrease rapidly." There are two components to this definition: (1) decrease and (2) rapidly. An antonym could be opposite of either of these. The word "skyrocket" is an opposite of the first component; the word "dwindle" is an opposite of the second. Either one of these two distinct antonyms would be correct.
    - In such cases, you will only see the antonym to one of the components (or both simultaneously, like "increment" for the above example), so you will never have to decide between the two. However, knowing all components to a definition is clearly important to identifying all antonym possibilities.

  - Technique 1: Turn it into a Synonym Question by Predicting the Answer
    - *Step 1:* Define the word as precisely and clearly as you can. Make sure to take note of all connotations and all components of the definition that could have opposites. If the word has more than one meaning, start with only one or two obvious ones that have opposites.
    - *Step 2:* Without looking at the answer choices, come up with a simple antonym to the word on your own (or 2 simple antonyms if you start with 2 definitions).
    - *Step 3:* Look at the answer choices and chose the word that is most similar to your prediction. In a way, you've turned an antonym question into a synonym question! If you do not find an answer choice that is similar to the one you predicted, the most likely possibility is that the question is using a different definition (especially if the answer choices are a

different part of speech or way off from your guess).

- If you do not find an answer, double-check that there is no antonym for the definition you are using. Look for an antonym for a different component of the same definition.
- If you don't see the answer when you double check, go back to Step 1 and start the process over with a different definition.
- If you don't know another definition for the stimulus word, then go on to Technique 2 since you are now working with a definition you are uncertain of.

- <u>Technique 2: For when you have familiarity but no precise definition</u>
  - *Step 1:* Use your approximate vocabulary techniques to get as much information about the word as possible. Then come up with a broad guess of what would qualify as opposite to what you have come up with (e.g. if the word is positive, the antonym must be negative).
  - *Step 2:* Work backwards: move to the answer choices and form antonyms for them.
  - *Step 3:* Eliminate any words without clear antonyms.
  - *Step 4:* Choose the answer whose antonym seems most similar to what you think the stimulus word means.

- <u>Technique 3: For when you are dealing with a completely unfamiliar word</u>
  - Use your approximate vocabulary techniques to get as much information about the word as possible (as in Technique 2).
  - Work backwards (as in Technique 2).
  - Eliminate any words without clear antonyms (as in Technique 2).
  - If you have some sense of what the stimulus word means then choose the answer whose antonym seems most similar to what you think it means.
  - If you still have no idea, or are torn between a few choices still, chose the most extreme answer choice that you have left.
    - Extreme words are more likely to be correct than moderate words.

ANTONYM ANALYTICAL CHECKLIST

- Technique 1: Turn it into a synonym question
- Technique 2: When you have familiarity but no precise definition
- Technique 3: When you have no familiarity

DAY 3
TABLE OF CONTENTS

- Quadratic equations and how to solve for their roots
- Familiar quadratics

DATA ANALYSIS

12. General approach and description

ARITHMETIC II

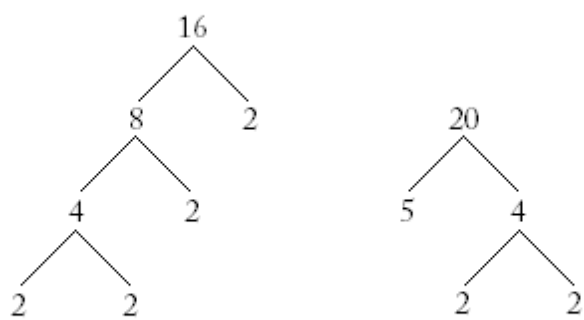
Fractions, Decimals, & Percents

1. Fractions

- Reduction Fractions: this consists of pulling out common factors of the numerator and denominator. For example: in the fraction 8/10, 2 is a common factor of both the numerator (8) and the denominator (10). Dividing 2 out yields 4/5.
    - One easy and reliable way to reduce fractions is to pull out the prime factors (using a factor tree) of both the numerator and denominator and cancel out corresponding factors.

Example: Reduce the fraction 16/20.

First get the prime factors (factor tree) of both the numerator and denominator and re-write the fraction:



$$\frac{2 \times 2 \times 2 \times 2}{2 \times 2 \times 5}$$

Next, cancel out like factors:

$$\frac{\cancel{2} \times \cancel{2} \times 2 \times 2}{\cancel{2} \times \cancel{2} \times 5}$$

Now multiply out what's remaining and we're left with our reduced fraction!

$$\frac{2 \times 2}{5} = \frac{4}{5}$$

The fraction thus reduces to 4/5.

Answer: 4/5

Example: Reduce the fraction 78/84.

Answer: 13/14

- Adding & Subtracting Fractions: to do this, the fractions need a common denominator. To add or subtract fractions with a common denominator, simply add or subtract the numerators (the denominator doesn't change). If they do not have a common denominator, you must change them so that they do.
  - The easiest way to find a common denominator is to simply multiply all the denominators together.
  - After you have done so, multiply each numerator by the *other* fraction(s) denominator(s). (Any common multiple would work for a common denominator, and here we recommend multiplying them because that will always create a common multiple).
  - The reason this works is because if you multiply the denominator by some term (let's say this term is "y") then multiply the numerator by the same term (again, "y"), then you have just multiplied the whole fraction by 1 (because y/y is just equal to 1), which doesn't change anything (since any number x 1 is that number).
  - Once you have a common denominator you simply add or subtract the numerators and put this answer over the common denominator. Reduce if necessary.
  - GENERAL FORMULATION FOR ADDING/SUBTRACTING FRACTIONS:

$$\frac{x}{y}+\frac{a}{b}=\frac{xb}{yb}+\frac{ya}{yb}=\frac{xb+ya}{yb}$$

  - BE CAREFUL: A common mistake that the GRE will try to exploit is adding the terms in the denominator. DON'T do this:

$$\frac{x}{y}+\frac{a}{b}\neq\frac{x+a}{y+b}$$

  *You cannot add numerators if they are not over a common denominator
  *Never add denominators, they just "hang out"

Example: Add: 2/3 + 3/4 + 5/6.

Because these fractions all have different denominators (3, 4, 6) you must first create a common denominator by multiplying all of these together.

3 x 4 x 6 = 72

We see that 72 is one common denominator that we could use. But we can't simply change the denominators of the fractions as this would change their values. Whatever we do to the denominator we must also do to the numerator! Thus we multiply all numerators by the *other* denominators:

2(4 x 6)/72 + 3(3 x 6)/4 + 5(3 x 4)/6 = 48/72 + 54/72 + 60/72

Notice that we have only changed the form of the fractions, and not their values: 48/72 = 2/3; 54/72 = 3/4; and 60/72 = 5/6.

Now, add all the numerators while maintaining the common denominator:

$$\frac{48 + 54 + 60}{72} = 162/72$$

Reduce the fraction and change to a mixed number if needed (which is likely because all GRE answers are in the most simplified form, which for a fraction is a mixed number):

162/72 = 9/4 = 2 1/4

Answer: 2 1/4

IMPORTANT NOTE: While this problem was solved using a method that will *always work*, you could have also created a much smaller common denominator if you realized that 12 is the lowest common multiple of 2, 4, and 6. Then the problem would have looked like this:

2/3 + 3/4 + 5/6
8/12 + 9/12 + 10/12 = 27/12
27/12 = 9/4 = 2 1/4

The important thing to notice here is that however you change the denominator, you must do the same to the numerator:

To change the denominator of 2/3 to 12, we have to multiply the denominator by 4 (3 x 4 = 12). So the numerator also has to be multiplied by 4 (2 x 4 = 8). The same method applies to the other fractions. To get from 4 to 12 we multiply by 3, so we also multiply the numerator by 3, giving us 9/12. To get from 6 to 12 we multiply by 2, so we also multiply the numerator by 2, giving us 10/12.

- Multiplying Fractions: Because fractions are based upon division (they can also be represented as the numerator divided by the denominator), multiplying fractions is much more straightforward than adding or subtracting fractions.
  - There's nothing tricky about multiplying fractions. Simply multiply straight across: numerator x numerator and denominator x denominator.

Example: Multiply: 2/3 x 5/12.

Just multiply straight across:

$$\frac{2}{3} \times \frac{5}{12} = \frac{10}{36}$$

Now reduce via prime factoring and canceling:

$$\frac{\cancel{2} \times 5}{\cancel{2} \times 2 \times 3 \times 3} = \frac{5}{18}$$

Answer: 5/18

- Dividing Fractions: Dividing by anything (not just fractions) is the same as multiplying by the reciprocal. Recall from Day One: flip the numerator and the denominator of a fraction to find its reciprocal.
  - Thus, when you are dividing fractions, simply flip the divisor and then multiply.

Example: Divide: 2/3 ÷ 4/5.

Start by inverting the second fraction:

4/5 becomes 5/4

Then multiply normally:

2/3 X 5/4 = 10/12

And simplify:

10/12 = 5/6

Answer: 5/6

Now it's your turn:

Example: Divide: 12/18 ÷ 6/10 =

FRACTION ARITHMETIC GENERALIZED FORMULAS	
ADDITION & SUBTRACTION	$\frac{x}{y}+\frac{a}{b}=\frac{xb}{yb}+\frac{ya}{yb}=\frac{xb+ya}{yb}$
MULTIPLICATION	$\frac{x}{y}\times\frac{a}{b}=\frac{xa}{yb}$
DIVISION	$\frac{x}{y}\div\frac{a}{b}=\frac{x}{y}\times\frac{b}{a}=\frac{xb}{ya}$

Answer: 10/9

REMEMBER: any integer can be turned into a fraction by putting it over 1.

2. Decimals

- Decimals: a decimal is any number written with a decimal point. All numbers can be expressed as decimals, including fractions (e.g. 1.5, .25, 10.3, etc.).
  - To turn fractions into decimals, divide the numerator by the denominator.
    - You can do this using long division even when the denominator is bigger than the numerator. All you need to do is add a decimal point with 0s after the number (e.g. 3 → 3.0000). Then put the decimal point in the same place above the division symbol.
  - To turn decimals into fractions, take all of the numbers to the right of the decimal place as a single number and multiply it by the place value of the last digit (or divide by its reciprocal). Then reduce. Note that if there are any numbers to the left of the decimal then you will get a mixed number.

- Thus .75 would be converted by multiplying 75 by 1/100 because the 5 is in the hundredths column. (Keep in mind multiplying by 1/100 is the same as dividing by 100.) This would give you 75/100, which reduces to 3/4.

Example: Write 3/4 as a decimal.

First set this up as a long division problem:

$$4\overline{)3}$$

Clearly 3 is not divisible by 4, so we need to add a decimal point with extra 0s. We must also add a decimal point above the division sign in the *exact same place*.

$$\begin{array}{r} . \\ 4\overline{)3.0000} \end{array}$$

Then we just do long division as usual. 4 won't go into 3, so we put a 0 in the units column. It does go into 30, 7 times, so we put a 7 above the first 0 and we are off.

$$\begin{array}{r} 0.7 \\ 4\overline{)3.0000} \\ \underline{28} \\ 2 \end{array}$$

We have a remainder of 2, so we aren't done yet. Pull the 0 down to make it 20. 4 goes into 20, 5 times, so we put a 5 above the second 0.

$$\begin{array}{r} 0.75 \\ 4\overline{)3.0000} \\ \underline{28} \\ 20 \\ \underline{20} \\ 0 \end{array}$$

As soon as you get a result without a remainder you are done.

Thus 3/4 = .75.

Answer: .75

NOTE: If you can't get a result without a remainder it is probably because the fraction must be represented as an infinite repeating decimal. Put a line over the repeating pattern to represent an infinitely repeating decimal (e.g. for 4/3, which would be 1.333333…, the line would go over the final 3). If your fraction falls under this category, keep dividing until you can discern the repeating pattern (try converting 4/3 to see how the repeating 3 emerges).

Example: Convert .368 into a fraction.

Because 8 is the furthest digit to the right and is in the *thousandths* place, you will be multiplying by 1/1000 (or dividing by 1000). Drop the decimal and multiply by 1/1000.

368 x 1/1000 = 368/1000

Then reduce.

368/1000 = 46/125

Thus .368 = 46/125

Answer: 46/125

- Adding & Subtracting Decimals
  - To add or subtract decimals, line up the decimal points and add/subtract the same way you would whole numbers (remembering to bring down the decimal point!).
    - If the decimal is missing from a number you need to work with, simply add the decimal point and as many 0s you need to the right of the decimal point. Likewise, if you need more decimal places, add them in as 0s on the right.

Example: Add: 23.4 + 76 + 234.567 + 0.87.

Line up decimals and add, putting in extra 0s where needed:

```
   23.400
   76.000
  234.567
+   0.870
---------
  334.837
```

Answer: 334.837

- Multiplying Decimals
  - The only tricky thing to multiplying decimals is remembering where to put the final decimal point—other than that it's just good ol' long multiplication.
  - Start by multiplying the numbers as you would typical whole numbers.
  - Afterwards, count the number of digits to the right of the decimal point in each of your multipliers and add these up.
  - Place your decimal point in the final answer the same number of spaces from the right as the count of *all* digits to the right of the decimal points from *both of* the multiplied numbers.

Example: Multiply: 40.012 x 3.11.

Multiply as you would whole numbers, ignoring the decimals:

```
   40.012
X    3.11
    40012
    40012
+120036
12443732
```

Now count the number of decimal places to the right in the multipliers:

40.012 has 3 digits to the right of the point   3.11 has 2   3 + 2 = 5 total

Now take your result and move your decimal point the sum total of decimal points over from the right.

Answer: 124.43732

- Dividing Decimals: there are 2 techniques for dividing decimals. Which one you use depends on whether the decimal is the dividend or the divisor.
  - Technique 1: Decimal in the dividend (e.g. 11.25 ÷ 5): divide as usual, remembering to put the decimal above the division sign *exactly* where it is in the dividend.
  - Technique 2: Decimal in the divisor (e.g. 5 ÷ 11.25): divisors cannot have decimals, so you must do a conversion first and then divide.
    - To convert the divisor, move the decimal to the right as many places as needed to turn it into a whole number.
      - Then, move the decimal point in the dividend an *equal number of places*.
    - NOTE: You may need to add zeros to the dividend to accommodate for extra digit places.

Example: Technique 1 (Decimal in dividend). Divide: 11.25 ÷ 5

Set up the long division as usual, making sure to add a decimal point above the division line directly above the decimal point in the dividend.

```
    .
5)11.25
```

Then divide as usual.

```
   2.25
5)11.25
  10
   12
   10
    25
```

Answer: 2.25

Example: Technique 2 (Decimal is in the divisor). Divide: 5 ÷ 1.25

First, move the decimal to the right 2 spaces to make the divisor a whole number:

1.25 → 125.0

Now, move the decimal the same number of spaces (2) in the dividend:

5.0 → 500.0

Now, divide as usual:

500/125 = 4

Answer: 4

3. Percentages

- Percentages: percentages are decimals converted by a factor of 100. *Every* percentage is a decimal that has been multiplied by 100.
  - All percentages are ways of writing a fraction with a denominator of 100 (percent literally translates to "per 100").
  - To convert a decimal to a percentage, *multiply* by 100 and add the "%" sign (.63 → 63%). To convert a percentage to a decimal, *divide* by 100 and remove the "%" sign (63% → .63).
  - Any time you encounter a percentage, whether the problem says it in words or denotes it with the % symbol, the number you are looking at has already been multiplied by 100.
    - BEWARE: The GRE questions will use misleading numbers to exploit any possible confusion with this. For example, you may see a QC problem asking you to compare .9% and .07. This is a trick because the % on the .9% means it has already been multiplied by 100, and would thus be .009 in decimal notation, which is clearly less than .07 (which would be 7%)!

- Finding percentages of numbers
  - To determine the percent of a number, change the percent you're looking for to a fraction (over 100 of course) and multiply.
  - Alternatively, you could also slide the decimal point left 2 spaces (which is equivalent to dividing by 100), and then just multiply by this decimal.

Example: What is 35% of 80?

Turn the percent into a fraction and multiply (or slide the decimal left 2 spaces and multiply)

35/100 x 80 = 2800/100 (or .35 x 80 = 28)

If you use the fraction method, you need to divide the 100 out to get the answer. (As you can see above, this step is unnecessary if you use the decimal method).

2800/100 = 28.

Answer: 35% of 80 is 28.

- <u>Finding percentages *from* numbers</u>
  - The last section shows you how find the percentage *of* a number. Finding a percentage *from* a number is similar but slightly different. You will use the same equation, but here you know the two numbers instead of one number and a percentage. (In the above example, this would mean needing to find what percentage 28 is of 80; in other words, you would know 80 and 28 but not 35%, which is what you would be solving for.)
    - If you are given the values of numbers, you must divide one number by the other and then *multiply by 100*. The denominator in the division will be the "percent of" number. Be careful: this is often—but not always!—the bigger number. For example, 250 is 200% of 125: something you could only figure out if you divide 250 by 125.
      - In order to avoid mistakes, double-check that your solution makes sense. If the question is "What percent of y is x?" and x > y, the percent must be greater than 100%. Conversely, if the question is the same but x < y, the percentage will be less than 100.
      - Do not forget to multiply by 100 after you are done dividing! This is an easy step to forget when you are under pressure.

Example: What percent of 400 is 15?

Divide first.

15/400 = 3/80 = .0375

Now multiply this decimal by 100 to convert it to a percent.

.0375 x 100 = 3.75

Answer: 3.75%

Example: 36 is 12% of what number?

The equation to solve this should look like: .12x = 36

Answer: 300

- <u>Percent Increase/Decrease</u>: these are tricky because they are asymmetrical: the percent increase from one number to another is not equal to the reverse percent decrease.
  - To find the percentage by which something has increased or decreased, use the following formula:

Percent Change = (Difference/Original) x 100

$$\%\ change = \frac{\Delta x}{x_{initial}} \times 100$$

  - The equation is the same for both percent increase and percent decrease; only the "initial" number changes. This is because in an increase the original number must be the smaller of the two and in a decrease it must be the larger of the two. This is why you get different percents for increase and decrease from the same numbers.

Example: What is the percent decrease from 18 to 12?

The difference between 18 and 12 is 6. The original number would have been 18 since it is a decrease.

% decrease = (6/18) x 100 = .333 x 100 = 33.333

Answer: 33.3%

Note that % increase from 12 to 18 is *not* 33.3%. This is because our difference would still be the same, but our original number is now 12, yielding a % increase of (6/12) x 100 = 50%.

- You should memorize the following fraction-decimal-percent relationships:

Fraction	Decimal	Percent
1/100	.01	1%
1/50	.02	2%
1/20	.05	5%
1/10	.10	10%
1/8	.125	12.5%
1/5	.20	20%
1/4	.25	25%
1/3	.33	33%
1/2	.50	50%

If you see a related fraction with a different number in the numerator (e.g. 3/10), multiply the decimal value (or percent) by the number in the numerator (e.g. 3/10 = 3 x .10 = .30 or 30%).

4. Exponents

- Exponents: exponents signify how many times a term gets multiplied by itself. The term is called the base. Exponents are represented in small print above and to the right of the base. In the expression 4^3, 4 is the base and 3 is the exponent. The value of the exponent (also called the power) is the number of times the base gets multiplied by itself. For example, 4^2 means 4 times itself 2 times (4 x 4); 4^3 means 4 times itself 3 times (4 x 4 x 4).

- Multiplication rules for exponents
  - Same bases: when multiplying terms with the same bases, simply add the exponents.

$$a^x \times a^y = a^{x+y}$$

    - The logic of this rule becomes clear if you expand the terms. Take 3^2 x 3^3 for example. This could also be written like this: (3 x 3) x (3 x 3 x 3). Parentheses don't matter for multiplication, so you could rewrite this expression like this: (3 x 3 x 3 x 3 x 3). Count 'em: that is 5 3s multiplied there, which is 3^5, just as the rule states. This logic holds true for the subsequent rules as well (although we will not be writing all of them out).

      REMEMBER: You will be learning all of these formulas in one direction, but keep in mind they work both ways! For example, you could also take the 3^{11} below and break it up into 3^4 x 3^7. The folks that write the GRE love to make you think outside the box and use formulas "backwards."

Example: What is 3^4 x 3^7?

The base is the same (3) so just add the exponents:

4 + 7 = 11

So:

3^4 x 3^7 = 3^{11}

Answer: 3^{11}

REMEMBER: This only works when the bases are the same.

- Same exponents: if the terms have the same exponents, multiply the bases and keep the same exponent.

$$a^y \times b^y = (a \times b)^y$$

  - Keep in mind that this means that terms such as $(3x)^2 = 9x^2$—*not* $3x^2$—because you must square *both* terms.

- Division Rules for Exponents
  - Same bases: as with multiplication, when you are dividing terms with the same bases, all you need to do is subtract the exponents and maintain the base.

$$\frac{a^x}{a^y} = a^{x-y}$$

Example: What is $6^5 \div 6^2$?

The base is the same (6) so just subtract the exponents:

5 – 2 = 3

So:

$6^5 \div 6^2 = 6^3$

Answer: 6^3

REMEMBER: As before, this only works when the bases are the same.

- Compound Exponents: these are terms with exponents both inside and outside parentheses ("a power to another power"). Multiply the exponents (keeping the base of course).

$$\left(x^a\right)^b = x^{a \times b}$$

Example: What is $(5^3)^2$?

Multiply the exponents and keep the base:

3 x 2 = 6

So, keeping the base, we get:

$(5^3)^2 = 5^6$

Answer: 5^6

GENERALIZED EXPONENT FORMULAS	
MULTIPLYING TERMS WITH SAME BASE	$x^a \times x^b = x^{a+b}$
MULTIPLYING TERMS WITH SAME EXPONENT	$a^y \times b^y = (a \times b)^y$
DIVIDING TERMS WITH SAME BASE	$\frac{a^x}{a^y} = a^{x-y}$
COMPOUND EXPONENTS	$(x^a)^b = x^{a \times b}$

- Other Exponent Rules
  - Any number with an exponent of 0 equals 1.
    $2^0 = 1$
    $6^0 = 1$
    $2{,}879^0 = 1$
  - Any number with an exponent of 1 is equal to that number.
    $2^1 = 2$
    $2{,}879^1 = 2{,}879$
    $-6^1 = -6$
  - When dealing with a negative exponent, raise the base to the exponent as you would if it were positive, then put your entire answer under "1" in a fraction (it's a reciprocal).
    $4^{-2} = 1/4^2 = 1/16$
  - When raising a fraction by an exponent both the numerator and denominator must be raised to the exponent.
    $(2/3)^2 = 2^2/3^2 = 4/9$
  - When dealing with a fraction as an exponent, raise your base to the value in the numerator position and take the denominator root of the answer.
    $$x^{\frac{n}{d}} = \sqrt[d]{x^n}$$

5. Roots

- Roots: a root is some term that, when raised to the indicated exponent, will equal a certain product. Think of them as "anti-exponents" in the sense that they signify the reverse of their analogous exponent. The symbol for roots is the radical sign: $\sqrt{\ }$.

- <u>Square Roots</u>: the square root of a number signifies which number multiplied by itself equals the number you are trying to root.
  - Square roots are signified by a root sign with no number to the left of the sign. They are in a sense the "default root," much like 1 could be thought of as the "default exponent" since it is not indicated when a number is raised to the power of 1.
  - The GRE does not test square roots of negative numbers because those are imaginary numbers: the GRE only tests real numbers.
  - Because the GRE only tests real numbers, for its purpose square roots <u>only have one solution</u> (the positive one), in contrast to squares, which always have <u>two solutions</u>.

Example: $\sqrt{16}$

This means "What number times itself equals 16?"

We know 4 x 4 = 16, so the square root of 16 is 4.

Answer: 4

REMEMBER: If $x^2 = 16$ then x can be either +4 or -4 (as both square up to 16). BUT on the GRE square roots are always positive, so the square root of 16 is *only* +4.

- <u>Approximating square roots</u>: finding the root of a perfect square is pretty easy, but if you need to find the root of an IMPERFECT square (a square of a non-integer), you will have to approximate. (The GRE will never require you to calculate exact values for imperfect squares.) Here's how:
  - First, find the *closest smaller perfect root* and the *closest larger perfect root* of whatever number you are trying to root. The answer must be between those.
  - Next, find which of the two bookend perfect roots your number is closer to and by how much. Use this to draw your approximation even closer to a correct answer.

Example: Approximate $\sqrt{32}$.

First let's find some perfect square bookends. The closest smaller perfect root is 25 and the closest larger perfect root is 36. So:

$$\sqrt{25} < \sqrt{32} < \sqrt{36}$$

Thus:

$$5 < \sqrt{32} < 6$$

To get an even better approximation, check where 32 sits between those bookends: it is closer to 36 than 25. So our approximation should be closer to 6 than 5:

Answer: ≈ 5.7

In fact the real answer is 5.66. Keep in mind this method only helps us find *approximate* values.

- Cube Roots: also called third roots, these are just like square roots, except they signify which number multiplied by itself *three times* equals the number you are trying to root. They are written with a small three outside the radical symbol to the upper left: $\sqrt[3]{x}$.

- Extended Roots: using an extended root of a number is essentially the opposite of using larger and larger exponents (just as a square root is the opposite of squaring). They are named after the value of the root as the "4th root," "5th root," "6th root," etc.
  - Roots greater than 2 must have their value written on the left of the radical symbol like so: $\sqrt[n]{x}$. This symbolizes the n^{th} root of x.
  - Z^4 means (Z x Z x Z x Z). Conversely, the 4^{th} root of Z ($\sqrt[4]{z}$) means, "Which number (x) multiplied by itself 4 times equals Z?" If $x^4 = Z$, then the value of $\sqrt[4]{z}$ would be x.
  - Z^5 = (Z x Z x Z x Z x Z). The 5^{th} root of Z ($\sqrt[5]{z}$) means, "Which number (x) multiplied by itself 5 times equals Z?" (In other words: $x^5 = Z$).

Example: What is the 4^{th} root of 16?

In other words, what number times itself 4 times equals 16?

We know $2^4 = 16$. So the 4^{th} root of 16 is 2.

Answer: 2

- Roots and Fraction Exponents: roots and exponents are related, and the way in which they are related has to do with fractions as exponent values. Fractions as exponents signify both the power the number is being raised to as well as the root that it has.
  - The numerator of a fraction in the exponent is the power that number is being raised by.
  - The denominator of a fraction in the exponent is the value of the root. Thus $\sqrt{x}$ is equivalent to $x^{1/2}$, $\sqrt[3]{x}$, is equivalent to $x^{1/3}$, and so on.
  - As stated before this means: $\sqrt[y]{a^x}$ is equivalent to $a^{x/y}$.

6. Root Arithmetic

- Multiplying Roots: numbers under radicals can be multiplied together as long as they are under the same root. This means that when roots are multiplied together, they can either be expressed individually or all under one radical. Thus: $\sqrt{6} \times \sqrt{8} = \sqrt{6 \times 8} = \sqrt{48}$.

$$\sqrt{x} \times \sqrt{y} = \sqrt{x \times y}$$

AND

$$\sqrt{x+y} = \sqrt{x} + \sqrt{y}$$

- Dividing Roots: roots can also be divided. Different situations will require you to use this in different directions. For example if you had $\frac{\sqrt{8}}{\sqrt{2}}$ you would want to convert this to solve as follows:

$\sqrt{\frac{8}{2}} = \sqrt{4} = 2$; However if you had $\sqrt{\frac{9}{36}}$ you would want to convert the opposite way to solve as follows: $\sqrt{\frac{9}{36}} = \frac{\sqrt{9}}{\sqrt{36}} = \frac{3}{6} = \frac{1}{2}$

$$\frac{\sqrt{x}}{\sqrt{y}} = \sqrt{\frac{x}{y}}$$

AND

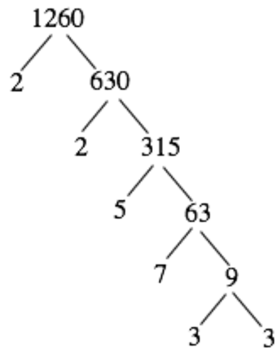
$$\sqrt{\frac{x}{y}} = \frac{\sqrt{x}}{\sqrt{y}}$$

- Adding and Subtracting Roots: you cannot add or subtract numbers under separate radicals, just as you cannot add or subtract exponents: $\sqrt{x} + \sqrt{y} \neq \sqrt{x+y}$. Nor can you split them up: $\sqrt{x+y} \neq \sqrt{x} + \sqrt{y}$. However, if *and only if* the radicals have the same base, you can add the coefficients.
  - $4\sqrt{3} + 7\sqrt{3} = 11\sqrt{3}$.
  - However, the expression $5\sqrt{2} + 3\sqrt{5}$ cannot be manipulated further!

- Simplifying Square Roots: to simplify, pull out all perfect squares from under the radical. If the number under the radical can be factored into only perfect squares, then you will end with just an integer. If it cannot, you will end with a "mixed root" (e.g. $2\sqrt{3}$). This expression is equal to the integer times the root (e.g. $2 \times \sqrt{3}$).
  - To simplify numbers under a radical sign:
    - Factor the number into its prime factors (prime factorization).
    - Pull out all the factors that are pairs because these are perfect squares. When you pull out the pair, only one value goes outside the radical. This is because the square root of a number times itself is that number ($\sqrt{2 \times 2} = 2$). All you are doing is pulling out all perfect squares.
    - Multiply the numbers you pulled out and put the product on the left of the radical.
    - Multiply the remaining numbers under a radical sign.
      - A helpful analogy: think about simplifying fractions into mixed numbers. Simplified roots often look similar to mixed numbers, and they follow a similar logic. When you are simplifying a fraction that is already reduced, you pull out all of the full integer values (e.g. 9/4 simplifies to the mixed number 2 1/4). Similarly, when you are simplifying roots you pull out all perfect squares (e.g. $\sqrt{8}$ simplifies to the "mixed root" $2\sqrt{2}$).
    - Note: if you needed to simplify a cube root, you would pull out the factors that occur in sets of 3, not 2 (since you would be looking in this case for perfect cubes); if it were a 4th root you would pull them out in sets of 4, etc.

Example: Simplify $\sqrt{1260}$.

First you do prime factorization for the number under the radical:



Prime factors of 1260 are 2, 2, 5, 7, 3, and 3.

Find all prime factors that can be paired, since these are your perfect squares:

There are two 2s and two 3s, so one 2 and one 3 will be pulled out front.

Then rewrite the term with the factors out front and in the middle and multiply:

$2 \times 3\sqrt{5 \times 7} = 6\sqrt{35}$

The simplified form of $\sqrt{1260}$ is $6\sqrt{35}$

Answer: $6\sqrt{35}$.

NOTE: The reason this works is because a number, such as 1260, can also be represented as the product of its factors, which here would be $2 \times 2 \times 3 \times 3 \times 5 \times 7$. Thus $\sqrt{1260} = \sqrt{2 \times 2 \times 3 \times 3 \times 5 \times 7}$, which according to the rules for multiplying roots can be rearranged to yield: $\sqrt{2 \times 2} \times \sqrt{3 \times 3} \times \sqrt{5 \times 7}$ From here it should be clear where $2 \times 3\sqrt{5 \times 7} = 6\sqrt{35}$ comes from!

Your turn:

Example: Simplify $\sqrt{75}$

Answer: $5\sqrt{3}$

REMEMBER: GRE answers will always be in their simplified form, so if you solve a problem and get $\sqrt{75}$, you will not see this in your answer choices, you will only see $3\sqrt{5}$.

7. Statistics for the GRE: Mean, Median, and Mode

- <u>Mean</u>: the mean of a group of numbers is simply the average of said numbers.
    - To find the mean, add the group of numbers and divide the total sum by the number of items.

$$Mean = \frac{Sum\ of\ items}{\#\ of\ items}$$

- The GRE will often ask you to use the formula for an average in ways that you might not be used to. For many of these problems it is very important to treat the sum of numbers as one term rather than a bunch of terms added together (similar to how you often want to solve for expressions rather than individual terms in a problem). Keep in mind that you can always find the sum of the terms by multiplying the mean by the number of terms.

$$Sum\ of\ items = (Mean)(\#\ of\ items)$$

Example: What is the mean of 10, 20, 35, 40, and 45?

Step 1: add up the numbers.

10 + 20 + 35 + 40 + 45 = 150

Step 2: divide by the number of terms. In this case there are 5 items, so divide by 5:

150 / 5 = 30.

Answer: 30

Your turn:

Example: Find the mean of 0, 12, 18, 20, 31, and 45.

Answer: 21

Example: A set of 8 numbers has a mean of 96. When one number is removed the mean becomes 93. What is the value of the number that was removed?

First you must solve for the sums of the two sets of numbers (don't forget that the number of items is 8 for the first set, but 7 for the second because one number was removed!):

96 X 8 = 768 This is the sum of the items for the first mean
93 X 7 = 651 This is the sum of the items for the second mean

The value of the number that was removed is just the difference in these values.

768 – 651 = 117

Answer: 117

TAKE NOTICE: While 117 is quite a bit bigger than the average (+ 21 to be exact), removing it did not have a huge impact on the average (which became smaller by 3). The impact of any individual number on an average is scaled by 1/(# of items). Notice that with 7 items a missing value that is 21 larger than the average only brings average down by 3 (21 X 1/7 = 3).

- Median: the median of a group of numbers is the middle number in the list after it has been written out in order from smallest to largest.

- If the set has an even number of items, the median is the average of the middle 2 numbers.

Example: What is the median of the list 9, 4, 6, 2, 15, 3, and 4?

Step 1: write the numbers out in ascending order.

2, 3, 4, 4, 6, 9, 15

Step 2: find the middle digit. In this case it is 4.

Answer: 4

Your turn:

Example: Find the median of 3, 125, 67, 32, 14, and 1.

Answer: 23 (This is the average of 14 and 32 because both 14 and 32 are the middle numbers.)

- Mode: the mode of a set of numbers is the number that appears most frequently.
    - It is possible to have more than one mode.

Example: What is the mode of 2, 5, 4, 2, 4, 3, 6, 9, 2, and 1?

Let's write the list out to help see the numbers better:

1, 2, 2, 2, 3, 4, 4, 5, 6, 9

From this list it's clear the most frequent number is 2.

Answer: 2

Example: What is the mode of 2, 9, 3, 4, 9, 3, 3, 5, 6, 9, 1, and 9?

Answer: 9

REMEMBER*: Each set of numbers has only ONE mean and only ONE median, but there can potentially be MANY modes!*

8. Probability, Permutations, and Combinations

- Probability: probability refers to the likelihood of a certain event occurring or not occurring.
    - Think of probability in terms of fractions. The probability of an event is just the number of instances of that event divided by the total number of instances.

$$\text{Probability} = \frac{\text{\# of possible outcomes that satisfy a certain condition}}{\text{\# of total possible outcomes}}$$

- If it is impossible for something to happen, its probability is equal to zero.
- If something is certain to happen, its probability is equal to 1.
- Everything else falls as a fraction in between zero and one.

Example: 15 marbles are placed in a bowl; some are red, all the others are blue. If there is one more red marble than blue in the bowl, what is the probability of reaching in and pulling out a blue marble?

Answer: 7/15

- Multiple Probabilities: if you are asked to determine the probability of two or more events occurring, first find the probability of each event occurring individually, then multiply all of those probabilities together.

  Example: If Jim rolls a normal die three times, what is the probability he will roll a two each time?

  First find the probability of each event.

  There is 1 two on a die, and 6 possibilities, so the chance of each individual event is 1/6

  Then multiply the probabilities of all events together.

  1/6 X 1/6 X 1/6 = 1/216

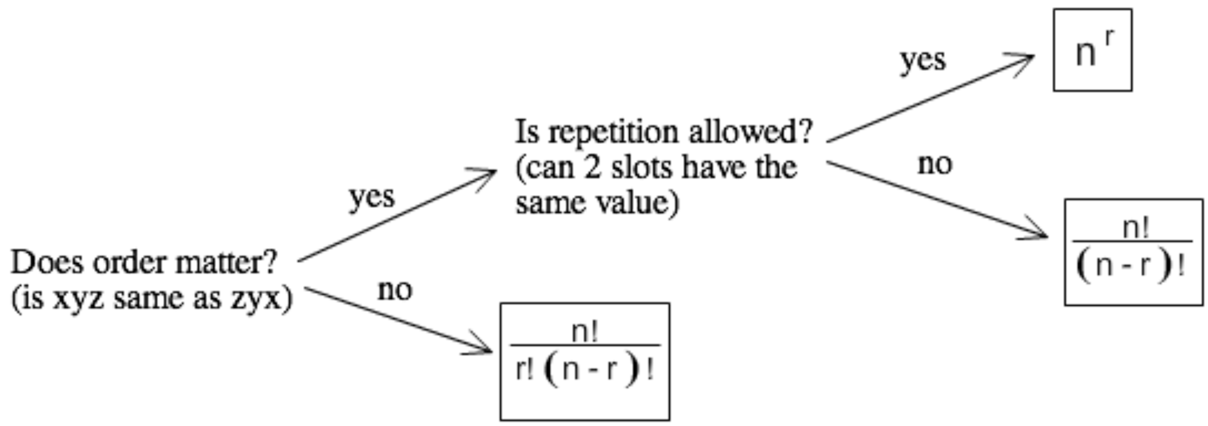
  Answer: 1/216

  - If you are asked for the probability of one event OR a different event occurring. In this case, rather than multiplying the individual probabilities together, add them.

    Example: If Timothy selects a playing card from a typical 52 card deck, what is the probability he will select a 4 or a 7?

    Answer: 1/26

- Permutations and Combinations: both permutations and combinations deal with the number of possible combinations given some set of possibilities, and the only difference between the two is whether order matters. It is unlikely you will encounter one of these problems on the GRE. If you do, just use the following decision tree and the appropriate formula. All formulas and decisions are explained below the tree.
  - In the following formulas *n* represents the number of options, and *r* represents the number of slots to fill.



- <u>Does order matter?</u> To answer this question you must figure out whether, given some options—say X, Y, and Z—arrangements with different orders are unique or the same (e.g. is XYZ the same as ZYX? Or are they different?).
  - For example, order would not matter if you were trying to determine something like options on a car (where AC, sun roof, and power locks is the same as power locks, AC, sun roof).
  - Some examples of when order matters would be how many ways you could order books on a shelf, knock pool balls in (where 16-15-14… is different than 1-2-3…), or set up the digits on a combination lock (where 9-8-7 is a different combo than 7-8-9).
- <u>Is repetition allowed?</u> To answer this question you must figure out whether values can exist more than once (i.e. can different slots have the same thing?).
  - Repetition would not be allowed for something like books on a shelf (one book can't be in 2 places at once), or, similarly, the order of knocking pool balls in.
  - Repetition would be allowed for something like a combination lock, which has the same 10 digit options for each slot (a combo could be 9-9-9).
  - Two of these equations have what are called <u>factorials</u> and are symbolized by an exclamation mark (x!). A factorial of a number is the number multiplied by every lesser integer between it and 0. For example 5! would equal 5 x 4 x 3 x 2 x 1 = 120.
  - Remember that operations in parentheses must be done first. So that (5 - 3)! would be 2!, not 5! - 3!

Example: You just bought 6 new plants to put in your garden. When you get home you realize that there are only 3 spots in the garden. How many different arrangements are possible given your 6 plants and the 3 spots to put them?

First you need to decide if order matters. Because rose-lilac-daffodil would be a different arrangement than daffodil-lilac-rose, order does matter, meaning this is a permutation problem.

Next you need to decide if repetition is allowed. Because one plant cannot occupy two spaces, repetition is not allowed.

Thus we use the formula $\frac{n!}{(n-r)!} = \frac{6!}{(6-3)!} = \frac{6\times5\times4\times3\times2\times1}{3\times2\times1} = 6\times5\times4 = 120$

Answer: 120

NOTE: When you have factorials in both the numerator and denominator it could take up quite a bit of your valuable and limited time to multiply them all out (especially with something like 9!). The trick to minimize the amount of computation is to cancel. 6! can be re-written as 6 x 5!, 6 x 5 x 4!, 6 x 5 x 4 x 3!, etc. Thus the calculation above could be re-written as $\frac{6 \times 5 \times 4 \times 3!}{3!}$, and we could cancel out the 3!s, which makes the whole thing collapse to 6 x 5 x 4—a single, shorter calculation. This technique is invaluable when dealing with large factorials!

9. Ratios, Proportions, and Rates

- Ratios: ratios are relationships of relative amounts between different related quantities.
  - Ratios (like fractions, decimals, and percentages) are yet another way of representing division.
  - Ratios can be written in several different ways:

    X:Y
    The ratio of X to Y
    X is to Y
  - The ratio of X to Y *must be written* as X:Y or X/Y. It is *incorrect* to represent it as Y:X or Y/X. You can think of this in terms of "of/to."
  - Ratios represent parts in relation to other parts, such as 10 pennies to 2 nickels (10:2) or 10 dimes to 4 quarters to 1 dollar (10:4:1).

Example: Flour, eggs, yeast, and salt are mixed by weight in the ratio of 11:9:3:2 respectively. How many pounds of yeast are in a 20-pound mixture?

In order to solve such a problem we must use the ratio to set up an equation. In this case we know that there are 20 pounds of this mixture, so the weights of all ingredients must add up to 20 pounds. In order to do this, create a problem that can solve for the "common unit": the unit (x in the below equation) that correctly divides the 20 pounds so that it can be distributed amongst the ingredients.

$11x + 9x + 3x + 2x = 20$ pounds
$25x = 20$ pounds
$x = 20/25 = 4/5 = .8$ pounds

What x represents above is the size of the elemental unit. It can then be multiplied by any ratio value to find the actual amount. So, now since we want an amount of yeast, which is represented by the 3 in the ratio, we must multiply x by 3.

$3x = 3(.8$ pounds$) = 2.4$ pounds

Answer: 2.4 pounds of yeast

- Proportions: proportions are relationships of equality between ratios. They are used to solve problems in which the relationship between quantities doesn't change, but the amount or value does.
  - Accordingly, simply deal with proportions the same way you would fractions: cross multiply, find common denominators, reduce, etc.

Example: If a 1 hour phone call costs $5.20, how much would a 20 minute call at the same rate cost?

First make ratios with all known values filled in and put variables where there are any unknown values. Then set the two ratios equal to each other. It doesn't matter where you put the different types in each ratio as long as similar units are in similar places (i.e., both time units are in the denominator, the numerator, or are on the same side of the equal sign). The only incorrect way to set up a proportion is to put the same type of units *diagonal* from each other (e.g., numerator on the left side of the equation, but denominator on the right side of the equation). Also make sure all units for the same pieces are the same.

$$\frac{\$5.20}{60\text{ min}} = \frac{x}{20\text{ min}}$$

Then cross-multiply and solve.

(20 min)($5.20) = (60 min)x

Answer: x = $1.73

- Rates: rates are a special kind of ratio where the numerator tells you the amount of some quantity (usually a number of objects or distance) that is created or covered in some unit of time, which is the denominator.
  - The general equation for rates is:

$$rate = \frac{\#}{time}$$

    The numerator is # because rates can be used for anything that is done with relation to time. For example, velocity is a rate of distance/time. Another common application of rates involves the production of something such as shirts over (that equation would be #shirts/time).
  - Just as with all ratios, knowing any two of the above values will allow you to find the third. The GRE will often require you to solve for the # or time given the rate and the other. So don't forget:

$$\# = (rate)(time)$$

AND

$$time = \frac{rate}{\#}$$

  - Rates can be difficult. You should always check your answer to make sure it makes sense.
  - A hint for checking your work: do the math on the units. You can do algebra and arithmetic on units just like you can on numbers and variables. For example, if you are solving for distance in miles your units should also work out to miles. The equation is d = rt which in terms of units would be miles = (miles/hour)(hour): the hour terms cancel, leaving you with miles, which is what you want.

Example: It takes Bill three hours to drive to work at an average speed of 50 miles per hour. If he takes the same route but averages 60 miles per hour, how long will it take Bill to get to work?

We know the typical rate (50 miles/hour), the typical time (3 hours), and the new rate (60 miles/hour). If we want the new time, we must first solve for distance since that is what connects the typical situation with the new one.

Distance = (rate)(time) = (50 miles/hour)(3 hours) = 150 miles

Now that we have the distance and the new rate we can solve for the new time.

Time = distance/rate = (150 miles)/(60 miles/hour) = 2.5 hours

Lastly, check to make sure your answer makes sense. If he is going a bit faster than usual it should take less time, not more. It checks out!

Answer: 2.5 hours

10. Simultaneous Equations

- Simultaneous Equations. In a situation in which you have more than one unknown value you will need more than one equation. You will need as many equations as you have unknowns. When solving word problems, for example, make sure you have enough equations. There are two techniques for solving multiple equations: the elimination method and the substitution method. Which method you choose to use should depend on which one is more convenient, but both will give you the same answer.

  - The Substitution Method for solving simultaneous equations involves solving one equation for one variable in terms of the other and then plugging that expression into the next equation.

  Example: If 2y – 4x = 6 and x + 2y = 26, then what does *x* equal?

  You have two equations, so pick one and solve it for the variable you *are not solving for*.

  2y – 4x = 6
  2y = 6 + 4x
  y = 3 + 2x

  Now just plug this expression in wherever you see *y* in the other equation and solve.

  x + 2(3 + 2x) = 26
  x + 6 + 4x = 26
  5x = 20
  x = 4

  Answer: x = 4

  NOTE: If you also wanted to find *y* you could now just plug the value of *x* into either equation and solve for *y*.

  - The Elimination Method involves doing arithmetic on the whole equations. You can do arithmetic on polynomials just as you can on monomials. To do this you will want to look for a variable that has the same coefficient in both equations so that you can eliminate it by either adding or subtracting the equations.
    - If the equations don't have a variable with the same coefficient in both, then you can always multiply one equation by anything you want to change the coefficients. However, as always, you must multiply everything in that equation by your conversion factor.

  Example: If x + 2y = 6 and 2x + 2y = 4, then what does y equal?

  Because we are solving for y we should try to eliminate x. Because x has different coefficients in both equations, we will need to multiply the first one by 2 to make them the same.

  2(x + 2y = 6)

$2x + 4y = 12$

Then we need to set up the equations so we can eliminate x. You can either add or subtract the equations, depending on which operation will eliminate x. In this case, because the term is positive 2x in both equations, we need to subtract them (because 2x – 2x =0). Line all the similar terms up and subtract down the columns.

$$\begin{array}{rr} - & 2x + 4y = 12 \\ & \underline{2x + 2y = 4} \\ & 0 + 2y = 8 \end{array}$$

Then solve.

$2y = 8$
$y = 4$

Answer: $y = 4$

- *Hint.* Often the GRE will ask you to solve for an expression rather than just a single term. In these cases, use the elimination method to create the expression rather than solving for each term first and then performing the operation. This is best seen with an example.

Example: If $5x + 4y = 6$ and $4x + 3y = 5$, then what does $x + y$ equal?

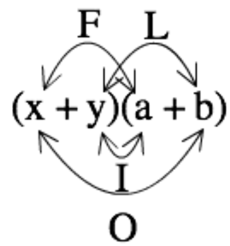
Because the question asks for the value of an expression "x + y," you should solve for this entire expression, NOT SOLVE FOR X AND Y INDEPENDENTLY. So you need to figure out how to get both terms to have a coefficient of +1 (since they both have a coefficient of +1). You should notice that $5x – 4x = x$ and $4y – 3y = y$, which is just what we want.

$$\begin{array}{rr} - & 5x + 4y = 6 \\ & \underline{4x + 3y = 5} \\ & x + y = 1 \end{array}$$

Answer: $x + y = 1$

11. FOIL, Factoring, and Quadratic Equations

- FOIL: multiplying polynomial expressions
    - When you multiply two polynomial expressions, remember that every term in the first set of parentheses must be multiplied by every term in the second parentheses.
    - This is where FOIL comes in. FOIL is an acronym for all of the individual multiplications that you must do to do the overall polynomial multiplication: **F**irst-**O**utside-**I**nside-**L**ast.
    - Each operation is illustrated in the following diagram:



- Let's see how this works: (x + y) x (a + b)
  - *First*: we multiply the first two terms: xa
  - *Outside*: we multiply the two outside terms: xb
  - *Inside*: we multiply the two inside terms: ya
  - *Last*: we multiply the two last terms: yb
  - Now you add them all together to get: xa + xb + ya + yb
  - Then simplify if needed.

Example: Multiply: (x + 4)(x + 2)

Let's FOIL. First:

$x \cdot x = x^2$

Outside:

$x \cdot 2 = 2x$

Inside:

$4 \cdot x = 4x$

Last:

$4 \cdot 2 = 8$

Now add everything up:

$x^2 + 2x + 4x + 8 = x^2 + 6x + 8$

Answer: $x^2 + 6x + 8$

Example: Multiply: $(4 - \sqrt{6})(4 - \sqrt{6})$

Answer: $16 - 8\sqrt{6} + 6$

- Factoring: to factor means to find two or more quantities whose product equals the original quantity. If you think about it, this is exactly what you are doing in prime factorization: finding the set of prime numbers that can be multiplied together to equal the number you started with.
  - There are many different ways to factor, prime factorization being just one of them. Moving from

elementary to complex, the next technique is pulling out common terms to simplify an equation. If you have an expression in which all terms share some coefficient (whether a number or a variable), you can pull this coefficient out and multiply the whole expression by it. This often helps you change the expression from an unwieldy form to something much easier to deal with.

- When doing this kind of factoring, you can always check your work by multiplying the term that you factored out back through to make sure you end up with the same thing you started with. When doing complicated factoring, such as with quadratics, always multiply back through to check your work!
- *Hint:* Often on the GRE you will have two expressions that are very similar, say in a numerator and a denominator, but not similar enough to be useful. Often factoring can help to get them in the same form, or at least a more usable form.

Example: Factor (xy + xz).

In this case, our common factor is x so we pull it out and re-write.

Answer: x (y + z)

Notice that if you multiply the x across the parenthesis, we end up with the same equation—our factor works.

- *Hint:* A good rule of thumb on the GREs: If you can factor, do it! The GRE will often make you integrate multiple tools to get to an answer. Knowing when to do so is part of the Tao of the GRE™. Many times, seemingly ugly problems will become simple once common terms are factored out. The key is recognizing when factoring is possible, because if you notice that something can be factored, it is probably a necessary step.

Example: Simplify: $\frac{(8^7)-(8^6)}{7}$

This looks tough, but remember: when in doubt, factor!
Tao hint 1: You don't know any formula for subtracting terms with exponents and a similar base.
Tao hint 2: You don't know any rules for dividing terms with exponents by a non-similar base Thus, you should be thinking how you can change this unruly form into something more manageable. You can factor an 8^6 out of the numerator to give you:

$$\frac{8(8^6)-8^6}{7}=\frac{8^6(8-1)}{7}=\frac{8^6(7)}{7}=8^6$$

(8-1) = 7. So we can cancel out our 7s, leaving just 8^6. This is an example of the way in which the GRE can give you expressions that are unfamiliar but are also just one step away from being simple. That is why it is a problem-solving test, not a math test! Listen to what the problem is telling you. If it is ugly, it is telling you that you need to change it into a clearer form.

Answer: 8^6

Your turn:

Example: Factor: $2y^3 - 6y$.

Answer: $2y(y^2 - 3)$

- Factoring Quadratic Equations: Reverse FOIL
  - The next level of factoring involves quadratic equations, which take the form $Ax^2 + Bx + C$, where A, B, and C are coefficients and x is a variable (for example $x^2 + 8x + 16$, and $3x^2 – 6x + 10$ are both quadratic equations). We explain quadratic equations in more depth below, but here we will show you how to factor them.
  - The easiest way to learn how to factor quadratics is to analyze them as FOIL in reverse. In order to do this, we will work through an example of FOILing an expression and analyze where all the terms came from. We will then use that analysis to show how you should think through factoring.
  - While factoring quadratic equations is not especially difficult, it is a little complicated, and warrants serious practice. With a little sustained practice you should be able to factor quadratics in your sleep, and this level of mastery is important for dealing with them on the GRE.

Demonstrative Example Part 1: Multiply the expression (x + 3)(x +4) using FOIL.

First: we multiply the first terms—(x)(x)—to yield x^2. This is where the Ax^2 term comes from. So, here A is just 1, but if A is not 1, its value will come from a coefficient on one or both of the x's.

Outer: then we multiply the outer terms—(x)(4)—to yield 4x. This *contributes* to the Bx term, but is not the whole thing.

Inner: then we multiply the inner terms—(3)(x)—to yield 3x. This also *contributes* to the Bx term, along with the product 4x above. In fact, the Bx term is the *sum* of the outer and inner products.

Last: then we multiply the last terms—(3)(4)—to yield 12. This is where the C term comes from: it is the *product* of the last terms.

Thus, we have $x^2 + 4x + 3x + 12$, which simplifies to $x^2 + 7x + 12$. Now we are going to factor this below.

Demonstrative Example Part 2 and How to Factor: Factor the expression $x^2 + 7x + 12$.

1. The first step in factoring quadratics is to dissect the Ax^2 term and set up our two expressions. Start by setting up two expressions in parentheses, each with an "x ± _", where in this case the "±" means either + or -, depending on the signs in the quadratic.

   (x ± _) (x ± _)

2. Next, we have to determine whether we need to use addition or subtraction. Because all terms are added in the quadratic, both expressions get + signs (we'll discuss other cases below).
   (x + _)(x + _)

3. Now we need to figure out what numbers go in those blanks. This is where understanding where the terms come from through FOIL is really helpful! Recall that the third term, C, is the *product* of the two numbers that fill the blanks. Therefore, in our example, one condition is that the numbers must be factors of 12. Also recall that the coefficient B in our second term is the *sum* of these numbers.

Thus, the other condition is that the sum of those two numbers must be B.

3a. First, you must factor C (in this case 12) to see what all of the possibilities are. The factors of 12 are: 1, 2, 3, 4, 6, and 12. These numbers satisfy condition one above.

3b. Second, you must figure out which two of those factors added together or subtracted yield B (in this case 7). The only two numbers that will yield 7 when added together are 3 and 4. As these are the only two numbers that satisfy both conditions one and two above, they go in the blanks!

The answer must be (x + 3)(x + 4).

4. Then, always FOIL your answer to double-check it. (x + 3)(x +4) = $x^2 + 7x + 12$!

- In the above example both of the signs in the parentheses were +'s, but they can also be –'s. Luckily, it is easy to know what signs to use by analyzing factoring as reverse-FOIL and remembering the multiplicative laws of positive and negative numbers. Here is a list of the relationships, but it would serve you well to derive these rules yourself to see how they work.
  - If the quadratic is of the form $Ax^2 + Bx + C$, then the signs in the factors will be +,+
  - If the quadratic is of the form $Ax^2 – Bx + C$, then the signs in the factors will be –,–
  - If the quadratic is of the form $Ax^2 – Bx – C$, then the signs in the factors will be +,–, with the smaller of the two numbers in the factors getting the + (since the B coefficient is the sum of these two numbers, the only way it will be negative is if the larger number is negative and the smaller positive).
  - If the quadratic is of the form $Ax^2 + Bx – C$, then the signs in the factors will be +,–, but the smaller of the two numbers in the factors will get the –.

Example: Factor: $x^2 – 3x – 10$.

This is a simple quadratic, so we first pull out x and put it in empty parentheses:

(x) (x)

Next we determine if the signs will be + or –. In this case we have $Ax^2 – Bx – C$, so one must be + and the other must be –, with the larger number getting the –.

(x +)(x –)

Now we find all factors of our C term, 10. These are 1, 2, 5, and 10.

Then we figure out which will have a difference (since we have one + and one –) of B (which is 3). The only two numbers with a difference of 3 are 2 and 5, so these must be our numbers. We then plug these in with 5 being in the second expression with the –.

(x + 2)(x – 5)

Last, you always want to check by FOIL-ing: $x^2 – 5x + 2x – 10 = x^2 – 3x – 10$. We're in business!

Answer: (x +2)(x – 5)

Your turn:

Example: Factor: $x^2 + 8x + 15$

Answer: $(x + 3)(x + 5)$

- Rarely, the A term in a quadratic will have a value larger than 1, as in the case of $4x^2 + 5x + 1$.
    - In the case of a valued coefficient in the A spot, find the two simplest factors, put them before x in the parentheses, and incorporate them in the remaining factor.
    - Before studying the below example, go back over basic quadratic factoring above. If the A coefficient is not 1, this adds another layer of complication to the factoring, so it is important that you understand how to do basic quadratic factoring in order to master this more complex type.

Example: Factor: $2x^2 + 10x + 12$.

In this case, the A coefficient is 2. So just add a 2 in before one x:

(2x + _) (x + _)

Now, on your own, factor the rest. The important thing to realize is that one of the numbers you put in will be multiplied by 2 when you add them up to create the 'B' in the middle Bx term. Once you figure this out, you should understand the full logic of quadratic factoring.

Answer: $(2x + 4)(x + 3)$

- Quadratic equations: above we mentioned that we'd delve into quadratics more in detail below—here we go! A quadratic equation is a polynomial equation that has some variable that is squared and no variables with exponents greater than 2.
    - It can be written as $Ax^2 + Bx + C = 0$, where A, B, and C are coefficients and x is a variable. This is the form that a quadratic formula must be in to solve for its *roots* (solutions) by factoring.
    - Roots: the solutions to a quadratic equation are called roots. These are the values of x that make the two linear expressions derived by factoring go to 0.
    - Every quadratic equation has 2 roots. These can sometimes be the same number, such that there is really only one numerical root. Just as squares (e.g. $x^2 = 4$) have two solutions (e.g. $x = 2, -2$), quadratics must as well.
    - You may remember from algebra that there are 2 ways to solve quadratic equations: factoring or using the quadratic formula. The GRE does not test the quadratic formula, so you will only need to know how to factor.
    - Typically the GRE will not use a larger exponent than 2 in this type of question, so don't sweat factoring larger exponents, just know how to factor and solve basic quadratics well!

- Solving quadratic equations:
    1. Push all the terms to one side of the = sign, leaving zero on the other side.
    2. Simplify all terms (i.e. add all x^2 terms together, all x terms together, and all constants together) to make the equation look like $Ax^2 + Bx + C = 0$.
    3. Factor (REMEMBER: this is just FOIL in reverse!).
    4. Set each factor equal to zero.
    5. Solve each individual factored equation (equal to zero).

6. Check by inserting your answers (the roots) into the original equation to make sure that they make the original equation go to 0 as well.

- Familiar quadratics: there are three quadratic expression the GRE loves to test. Memorize them!

$(x + y) \cdot (x - y) = x^2 - y^2$
$(x + y)^2 = x^2 + 2xy + y^2$
$(x - y)^2 = x^2 - 2xy + y^2$

Example: Solve: $x^2 - 6x = 16$.

First, put all terms on one side of the equation to set to zero.

$x^2 - 6x = 16$
$X^2 - 6x - 16 = 0$

Next, factor! Focus on the factors of the third term (16) that can be added or subtracted to make the coefficient of the second (6). In this case the factors of 16 are 1, 2, 4, 8, and 16, and we need 8 and 2.

$(x - 8)(x + 2) = 0$

In the same way that the product of xy would be 0 if *either* x *or* y is 0 because they are multiplied, this statement would be true if either $x - 8$ or $x + 2$ equals 0, and so we separate them, set each equal to 0, and solve.

$(x - 8) = 0$
$x = 8$

$(x + 2) = 0$
$x = -2$

The roots of this equation are 8 and -2.

To check, let's plug them back in to the original equation.

$x = 8$
$(8)^2 - 6(8) = 16$
$64 - 48 = 16$ ✓

$x = -2$
$(-2)^2 - 6(-2) = 16$
$4 - (-12) = 16$ ✓

Since both roots work for our original equation we know they are correct.

Answer: x = 8, -2

REMEMBER: Most quadratic equations will have TWO correct answers. All quadratics have two roots, and so the only way there can be only one answer is if both roots are the same.

DATA ANALYSIS

12. General approach

- The key strategy to DA: while many questions may require brief calculations, the ability to understand and siphon information quickly from a graph, chart, or table is key. In this sense, it's more a treasure hunt than a math quiz. READ THE GRAPHS BEFORE TACKLING THE QUESTIONS! When beginning the DA section, take some time to get acquainted with the graph(s) and figure(s). Know titles, scales, forms, etc. The more you understand before tackling the questions, the easier the questions will be.
    - The data analysis section is the one section on the quantitative part of the GRE in which *figures are drawn to scale*. This is very important to remember because you will often be asked to make estimates off of the figures. If you need to measure and compare something, don't just eyeball it. Put your scratch paper up to the screen and use it as a standard measurement tool by marking distances with your pen on the edge. This way you have a constant metric when comparing different elements on the figure(s).
    - There is NO REASON everyone shouldn't ace this section. The key to mastering the data analysis is working meticulously, making sure you know what is being represented and asked, and working through problems step by step (as they often require a few independent analyses). In other words: just pay close attention to what's going on and you'll do great!

- The kinds of graphs
    - Charts and tables: these organize data according to lists. To decipher the information presented, interpret the column and line headings, the numerical scale, and any overall descriptive title.
    - Bar graphs: these convert data into visual bars or columns. To decipher the information given, interpret the individual category names (listed on either the x or y axis), the scale used (listed on either the x or y axis), and any overall descriptive title. Look specifically for relationships between the bars presented.
    - Line graphs: these convert data into points on a grid connected by straight lines. To decipher the information presented, interpret the individual category names (listed on either the x or y axis), the scale used (listed on either the x or y axis), and any overall descriptive title. Look specifically at the slopes of the lines connecting the data points. These slopes reveal relationships through increases (positive slope) and decreases (negative slope).
    - Pie charts: these reveal relationships through percentages (and are also called circle graphs). The whole circle is 100% of whatever is being represented. The data is divided in the circle into separate parts representing different percentage amounts adding up to the whole. Don't forget that these are percentages: you will often be asked to convert these percentages into raw numbers based on some raw number that the figure or question gives you.

VERITAS TUTORS

DAY 4

TABLE OF CONTENTS

1. Sentence Completion

The question. Sentence completion questions present a sentence with one or two missing words. On sentence completion questions with one blank, you will choose from a list of 5 words or phrases. On the two-blank questions you will choose from a list of 5 word pairs.

The basics. Because the GRE will never present an open-ended question, everything you need to answer a sentence completion question MUST be contained in the sentence itself.

- Function words and sentence structure: the meaning of a sentence depends on two things: content and structure. Structure in particular is probably the most helpful analytical tool for sentence completion on the GRE.
  - Structure is the scaffolding created by function words, which you should analyze on every sentence completion task. Function words are words that do not have clearly definable meanings (unlike nouns, verbs, adjectives, etc.), but are the kinds of words whose role it is to create the skeleton of a sentence: they include conjunctions, logical phrases, and so on.
  - When you read a sentence on a sentence completion task, explicitly analyze the structure via its function words on the first pass. In order to do so, become acquainted with them:

    Some function words that express continuity:
    - Thus
    - Similarly
    - Also
    - And
    - Along with
    - Together with
    - Moreover
    - Indeed
    - As well as
    - In other words…
    - In addition
    - Additionally
    - Similarly
    - Just as
    - Comparable to
    - Besides
    - Therefore
    - As
    - Furthermore
    - Likewise
    - ; (semicolon)

Example: This law is concerned only with the specific time period in question; *therefore*, no attempt has been made to __________ phenomena _________ to that era.

A) include…unrelated
B) omit…irrelevant
C) re-create…germane
D) discuss…essential

E) evaluate…pertinent

When you first read this sentence you should notice that "therefore" links two pieces of it. This function word expresses a relationship of continuity between the two such that the second must agree with the first. This is the most important thing to know when evaluating answers: it means that the words you choose for the second part must make it consistent in meaning with the first part.

The first part says "concerned only with specific time period," so the second part must reflect that as well. The only answer choice that makes the second half parallel this is A.

Answer: A) include…unrelated

Some function words that express contrast:

- Despite
- However
- Although
- While
- Though
- Even though
- Notwithstanding
- However
- In contrast
- Nevertheless
- As opposed to
- Nonetheless
- Unfortunately
- Except
- On the other hand
- Instead
- But
- Despite
- Regardless
- Rather than
- Yet

Example: Most candidates spend ______ they can raise, *but* others wind up on Election Day with a _______.

A) so…bankroll
B) time…vacation
C) everything…surplus
D) every cent…deficit
E) nothing…war chest

Answer: C) everything…surplus

- Definitional Structure
  - Often sentences will be constructed in a way that makes your task more like finding the right word for a definition. When a sentence uses its structure to present you with a restatement of the

word, recognize that and look for the word that most closely matches the definition.

Example. The world of cinema is _______, both familiar and unrecognizable.

A) contradictory
B) realistic
C) simplistic
D) timeless
E) unchanging

The structure of the sentence makes "both familiar and unrecognizable" a restatement of the meaning of the word in the blank. So, find the word that most closely matches this definition: A) contradictory.

Answer: A) contradictory

- <u>Stand-alone sentences and argument structure</u>
  - One of the hardest parts of sentence completion tasks is that after eliminating obviously incorrect choices you are still left with more than one answer. You feel like the remaining choices all make "good sentences," and that you have no further criteria with which to distinguish between them. This can give the impression that the question is subjective, and therefore choosing the right answer is arbitrary and based on luck. This is where understanding argument structure and being able to dissect it can help.
  - Conceptually, what you need is not just an answer choice that generates a "good sentence," but also an answer that can "stand alone." One way to think about this is to ask yourself whether the sentence is the kind that would be good only if it were in the middle of a paragraph but cannot stand on its own. If that is the case, you can eliminate that choice. There may be more than one answer choice that makes the sentence a perfectly good and sensible sentence. However, there will only be one answer choice that makes a good sentence AND makes a good sentence that can stand on its own.
    - This is one of the main reasons why you should not add any of your own information to the sentence. Typically when we read we 'fill in the blanks' to help the writer get his or her message across. DO NOT HELP the sentences in the sentence completion. If you don't provide them with any support by adding information that isn't already in the question, you will always be left with only one answer choice standing.
  - A concrete way to analyze a sentence analytically and know whether or not a sentence can "stand alone" is to pull apart the argument structure. This technique offers a non-subjective method for those tough sentence completion questions in which you must choose between multiple "good sentences." To implement this method you will need to know about a few components of argument structure including: premises, conclusions, and function words that introduce them.
    - <u>Premise</u>: a premise is a declaration or proposition that is stated as a fact or assumption. Think of these as the evidence or starting points of an argument.
    - <u>Conclusion</u>: a conclusion is some piece of information that is supported by and derived from the premises. These are probably best understood though an example:

The classic illustration of argument structure is the following argument:

Premise: Socrates is a man.
Premise: All men are mortal.
Conclusion: Therefore Socrates is mortal.

The two premises are introduced as facts and need no backing. The conclusion is derived from these facts. This argument can also be expressed as a sentence: Socrates is a man, and all men are mortal, therefore Socrates is mortal.

In order for the argument to be complete, the premises must support the conclusion. If you were missing either premise, the argument would not be valid:

Socrates is a man therefore Socrates is mortal.
All men are mortal therefore Socrates is mortal.

Both of these sentences are missing a premise, and are therefore not valid arguments. The first sentence is lacking the premise "all men are mortal." Of course we all know this, *but it is not contained in the sentence*: if you used this knowledge you would be adding it to the sentence, which you should never do on the GRE! The same analysis holds for the second sentence.

- Thus, when you are left with more than one answer choice that makes a good sentence, you should immediately evaluate argument structure by identifying the premises and conclusion and checking to see if the sentence is missing a premise. Then eliminate all answer choices that create arguments with missing premises. Learning to do this well will give you an analytical and objective technique for difficult sentence completion questions.
  - In order to identify the premises and conclusions, look for function words that introduce them and dissect sentence structure to figure out what is being stated as a fact or assumption (premise) and what is being derived from these statements (conclusion).

  Function words that introduce premises:
  - Because
  - Since
  - Given
  - For
  - The fact that
  - Due to
  - Owing to
  - Is supported by
  - Is proven by

  Note that often there is no word introducing a premise: you will know what the premise is based on its relation to the conclusion.

  Function words that introduce conclusions:
  - Thus
  - Therefore
  - So
  - It follows that
  - Hence
  - Consequently
  - For this reason
  - Demonstrates
  - Establishes that

Here too there is often no word that introduces the conclusion, but you will know what it is because it is the main statement of the sentence that everything else is directed towards.

- Technique 1: The Straightforward Method
  - *Step 1:* Read the sentence, paying special attention to the function words that give the sentence its structure.
    - Read the sentence noting continuation, contradiction, and definitional words.
    - Use the structure of the sentence to understand how different parts are related, defined, etc.
  - *Step 2:* Make a prediction of what you think goes in the blank(s).
    - After reading the WHOLE question and before looking at the answer choices, think of words you would plug in. The answer choices will try to sway you: prediction eliminates this power.
    - Don't make difficult predictions: keep your words simple and meaningful, just enough to recognize synonyms in the answer choices.
  - *Step 3:* Look at the answer choices and search for the word(s) that match your prediction.
    - Note: when dealing with two blanks, it's often easier to deal with one word at a time or the overall relationship (i.e. opposite vs. similar).
  - *Step 4:* RE-READ THE SENTENCE WITH YOUR CHOICE!
    - Don't select your answer till you've re-read it into the sentence. Does it still work in context? Then and ONLY then should you finalize your answer and move on.

- Technique 2: Analyzing Argument Structure
  - This technique is for sentences that are tough to crack and leave you with more than one answer via Technique 1.
  - *Step 1:* Identify the premises and conclusion in the sentence argument.
  - *Step 2:* Once you have premises and conclusions clearly specified, make sure that your answer choices generate a valid argument: one in which the conclusion is supported by the premises and does not require another premise that is not in the sentence. Alternatively, ask yourself if the answer choice makes a good sentence, but one that would have to be buried in the middle of a paragraph, and therefore can't "stand alone."

- Technique 3: For When You Don't Know the Answer Choice Word(s)
  - This technique is just like Technique 1, but you will have to use all of your approximate general vocabulary strategies to get a sense of what the words mean.
  - *Step 1:* When reading the sentence, identify high-level general relationships inherent in the sentence structure like positive/negative, agreement/contrast, etc. This information will help you employ the last approximate general vocabulary strategy: letting the question help you figure out what the words mean. Use this in conjunction with other approximate strategies to get your best guess on the word definitions.
  - *Step 2:* Eliminate common wrong answers.
  - *Step 3:* Make your best guess with what you have left and don't stress about it!

- Common Wrong Answers: As with all formulae, do not rely too much on these, but knowing them are helpful for quickly identifying what are likely to be sucker answer choices.
  - *The Half and Half:* in sentences containing two blanks, make sure you know the relationship between the two blanks. Often, the GRE will present answer choices where either the first or second word fits nicely, but the other doesn't. Don't fall for this: make sure the words work with each other in the appropriate way *as well as* fit the sentence.

- *The Opponent:* in a definition sentence, the GRE often presents answer choices rife with difficult vocabulary. Many times, the correct answer choice's antonym will appear. Make sure the word you chose isn't the opposite of the word you need.
- *The Temptress:* certain answer choices will use large, technical sounding words. (This is more common in questions of higher difficulty.) The hope is that you haven't studied your vocabulary and will fall for these juicy temptresses. How do we avoid this? Well, STUDY YOUR VOCAB! And on the test, first deal with the answer choices you do understand. Only move to the unknown after you've safely eliminated the known.

SENTENCE COMPLETION ANALYTICAL CHECKLIST

- Technique 1: The straightforward method
- Technique 2: Analyzing argument structure
- Technique 3: When you don't know the answer choice word(s)

2. Reading Comprehension

The Basics.

- Mastering the Reading Comprehension: The type of reading and analysis required for the GRE takes some getting used to. The advice here will sound simple and obvious, but once you start practicing you will see it is anything but. Often subtle alterations in everyday activities are much more difficult to achieve than learning something entirely new.
- The reading comprehension section is another paradoxical element of the GRE. You will feel pressured to race through the reading so you can get right to the questions, but this is the worst possible thing you could do. Taking your time with the reading, and taking notes will not only help you get the right answers but will save you time overall, by helping you cruise through the questions.
- The number one thing you can do is try to consciously apply the techniques covered here and become aware of when you slip back into old habits.
- You also need to practice extensively. Perhaps surprisingly, the best way to really understand the logic of the reading comprehension section is by doing the same passage and questions multiple times. Often, on the first pass, you will feel that the questions are subjective and arbitrary. You will struggle to answer them, and then when you do, you will find that your choice was wrong. This is because the reading comprehension requires a novel approach in a familiar domain, making it difficult to master. However, if you do reading passages a few times, later revisiting ones you struggled with before, you will often see what you didn't the first time. This will help you "peek under the hood" to see how the reading comprehension works. You will notice that all of a sudden you know what cues to look for, what a question is asking, and what logical structure the questions are based on. Try to focus on why you were getting questions wrong on earlier attempts, and identify what you could do instead. Only by becoming intimately familiar with the reading comprehension will you be able to accomplish the subtle perspective shift necessary for an effective approach that will help you master the reading comprehension.

- Passage Types
  - *Biological Sciences*: Passages about botany, medicine, or zoology.
  - *Physical Sciences*: Passages about chemistry, physics, or astronomy.
  - *Humanities*: Passages about art, lit, music, folklore, or philosophy.
  - *Social Studies*: Passages about history, econ, government, or sociology.
    - Don't worry if you get a passage that is from a domain you are *unfamiliar* with. In fact, these will probably be easier because you are less likely to bring in outside information. All

passages are designed so that anyone can understand them, and all of the information you will need to answer the questions is in the passage.

- Sometimes you will come across words that are unfamiliar in the passages. This is expected and is not a problem. You are not expected to know domain-specific terms (e.g., technical scientific words); again, the passage will have all the information you need. Don't get hung up on strange words, but don't just skim over them either. Take note when you come across some bizarre word. Pay attention to how it is used in the passage and what information is given.

- Do worry if you get a passage from some domain *you know well*. This seemingly paradoxical advice is due to the fact that you should never bring in outside information: you should rely only on the information in the passage. If you are reading a passage on subject matter that you are very familiar with, you need to be hyper-vigilant to ensure that you rely only on the passage and don't read anything into it that is not there.
    - In addition, often we have strong opinions in familiar domains, and tend to read passages on those subjects quite critically or overly supportively depending on how aligned we are with the author's viewpoint. DO NOT DO THIS! Read for content; do not read in any opinionated way. The GRE does not care what you think about a passage, only how well you can read and understand it!

- Passage Lengths
    - *Short Length* passages contain 18-25 lines.
    - *Long Length* passages contain 50-65 lines.

- Question Types: you will always see a mixture of the following question types for any passage.
    - *Global*: Identify a central idea or primary purpose.
    - *Detail*: Identify truths according to the passage.
    - *Inference*: Identify what the passage "implies." **Hint**: if it's not specifically in the passage, it's not going to be the right answer. Inference questions will only ask you about what can be directly inferred from the passage, and thus the inference needed is the minimal amount that is possible from the passage.
    - *Logic*: Identify the function of a sentence, phrase, or paragraph in the context of the whole piece.
    - *Vocabulary*: Define a word as used in the passage.

- Active Reading: this is the single most important thing for nailing the reading comprehension!
    - Typically when we read, we do so in a relatively passive manner. You must learn to read in a much more active way to succeed on the GRE. Don't read the passage—attack it! Listed below are a few features of typical passive reading *to avoid*:
        - Reading just for the gist. Usually when we read something, say a newspaper article, the details quickly slip out of our minds. All we are trying to get is the take-home message, and we gloss over the rest. The gist is important for the GRE, but you must capture the details as well.
        - Skipping over unfamiliar words. This is something you probably do without even realizing it. You get a sense of what the word means from your sense of what the sentence means. On the GRE, if you come across a word you don't know, take a second to try and figure it out, either from context, or from information presented near it. It is important that you understand everything from the passage. Often, the questions ask you to define such words based on the information in the passage (vocabulary questions).
        - Glossing over complicated sentences. Again, when reading for the gist, often we cruise over complicated sentences without even realizing it, because understanding it 80% works just

fine. Once again, this will not work for the GRE. If you do not understand EXACTLY what a sentence means, reread it. You may have to reread it multiple times. Break it into pieces, use the local context, and figure out precisely what the sentence means.
    - Varying attention. When reading we almost never pay complete attention to everything, and also are often not aware of the fact that we are not paying attention. This is probably the hardest part about learning to read for the GRE—we aren't even aware of exactly how unaware we typically are while reading! You need to pay attention to every single word, sentence, and phrase in the passage, as well as actively think about how all of these pieces fit together to get the author's argument across.
  - Now that we have seen what active reading is not, let's go over what it is.
    - The most important part of active reading is paying complete attention to every single word you read and understanding how all of the pieces fit into a hierarchical structure (words→sentences→paragraphs→passage).
    - You want to read and analyze every sentence making sure you know what it means and how it fits into the bigger picture. If you don't, read it again.
    - Read to understand (1) what the author is saying, (2) why he or she is saying that (e.g. to inform, to persuade, etc.), and (3) how he or she says it (e.g. use X, Y, and Z evidence to demonstrate point P).

- Take Notes. You should also take diligent notes along the way. This is the single most important piece of advice for the reading comprehension because it supports and reinforces all the other pieces, as well as providing you with an indispensible tool for the questions—your own notes. While this may seem like a waste of time at the moment, in the end it will save you time (not to mention help you get the right answers!). You will inevitably get questions about the overall message of the passage as well as ones about the specific details in the passage. The time that you use to take rigorous notes will be more than made up by saving time when answering these questions.
  - On your scratch paper you should take notes that are divided by chunks for each paragraph. That is, each paragraph gets its own block of notes.
  - Within each paragraph chunk you should take notes for (1) each claim, argument, or statement presented, (2) how each fits into the whole passage, and (3) the specific details, evidence, or examples used for support or elaboration.

- After Reading and *Before* you move to the Questions. When you have finished actively reading the passage and taking tedious notes, you will be tempted to go straight to the questions and not use up any more of your precious time. Don't! There are a couple of quick things you need to do before you get to the questions. Again, don't worry about time: it will be more than made up when you fly through the questions (and most importantly, you will get the right answers). In addition, it will help keep the questions and answer choices from subversively affecting your understanding of what you have just read.
  - Briefly summarize what exactly the passage was about.
  - State the theme of the passage as clearly and concretely as you can (this will usually be the answer to a question you will get as well).
  - Describe the tone (e.g. persuasive, informative, etc.).
  - Pull these all together. Think about what the main point or argument was, and what the author was trying to accomplish. Then think about all the smaller pieces, and what role they each played in the passage (e.g. were they evidence? History? Foils?).
  - OK, now you can go to the questions. Most likely by this point you have already answered the majority of them with the exercises here. You have also minimized the risks of adding or forgetting information and having your perceptions altered.

- The Best and Only Technique
  - *Step 1:* Read the passage actively, as explained above, and take meticulous notes.
  - *Step 2:* Before moving to the questions, go over the passage as explained above and clearly state its theme, arguments, and details, as well as its author's central objective.
  - *Step 3:* Read the question, making sure you understand EXACTLY what it's asking (what type of question is it?).
  - *Step 4:* Use lead words and line references to find the relevant part of the passage. If the question asks about something in the passage without giving line numbers, check your notes to figure out where this is in the passage and briefly reread it to refresh your memory.
  - *Step 5:* Read about 5 lines above & 5 lines below reference line/words.
  - *Step 6:* Formulate an answer in your own words before looking at the answer choices, and only then select the matching answer choice.

- Tips for Specific Question Types
  - Global Questions
    - Global questions often ask for the primary purpose of a passage:
      '*The main idea of the passage is…*'
      '*The author of this passage is primarily concerned with…*'
      '*Which of the following would be the best title for the passage?*'
    - You should already have answered all of these kinds of questions right after finishing the reading. Search for the answer that reflects what you have already come up with.
    - In the rare case where you don't already have the answer on hand, here are a few tips if you need to go back to the passage to try and find it:
      - Keep it somewhat general! Anything too specific couldn't encompass an overarching idea.
      - Look at the first sentence(s) of the passage, the last sentence of the first paragraph, or the conclusion: these are typically the best places to find answers for global questions.

  - Detail Questions
    - Detail questions ask about specific concrete details that will be explicitly laid out in the passage.
      - A good rule of thumb: If you can't place your finger on it in the text, it's probably not the right answer.
    - When referred to a specific line in the text, be sure to read above and below that line (in addition, of course, to the line itself). This will help you avoid the question leading you astray and will help put the line in context, which is everything when it comes to details.
    - If the question does not give lines specifically, check your notes and briefly reread the part of the passage that contains the relevant information.
    - Only go to the answers after you have checked the text, and already have a rough answer formulated in your head.

  - Inference Questions
    - While these questions do ask you to make an inference that goes beyond the passage, they are not going to ask you to go much beyond the text. Read the question and try to predict the answer. You should briefly try to construct an argument in your mind that clearly shows why that can be *safely* inferred from the text *using specific examples*.
    - Check the answers for your predicted inference. If you don't find it, then choose the one that you think is most reasonable and conservative. Before putting this answer in, make sure

that you can construct a clear and simple argument *based on concrete examples from the text* for why this is a valid inference.

- Logic Questions
  - Logic questions ask you to identify the purpose of a section in the passage or question the author's attitude/style.

    "*The author's tone can best be described as…*"
    "*The author most likely thinks the reader is…*"
    "*The author's intentions for adding paragraph 3 were…*"
  - While the questions will ask you to move a little bit beyond the passage, your answer should (as always) be clearly backed by evidence from the text. Before choosing any answer make sure you have constructed a brief, but clear and concrete mental argument based on the passage as to why your choice is correct.
  - A few good rules of thumb for these questions as well:
    - The GRE respects its authors and, therefore, will never have a correct answer choice disrespecting the writer or any other important figure.
    - Answer choice too negative or extreme? Get rid of it: the GRE typically avoids overly strong emotions or hot-button ideas.

- Rules of Thumb
  - Use common sense, but NEVER use outside knowledge. If it's not in the passage or cannot be clearly backed up by concrete examples from the passage, it's not right.
  - Too extreme? Then it's wrong.
    - *Common extreme words and phrases:* must, the first, the best, only, totally, always, no, all, every, each.
    - *As a foil, here are some less extreme, oft-used words:* may, can, often, sometimes, many, some.
  - What part of the passage are we talking about, and what is the specific question?
    - The GRE's favorite trick is to supply answer choices that do contain correct information pulled straight from the passage…but do not reference the part of the passage you were asked to consider. This is exactly what your notes are for.
    - ALWAYS make sure you're looking where you're supposed to be looking. Just because the answer is correct doesn't mean it's correct for the question at hand!
  - Half Wrong = All Wrong
    - **Remember**: Your job is to find flaws in the answer choices and eliminate accordingly.
    - The GRE loves to start answers out correctly but skew them incorrectly by the end. DO NOT FALL FOR THESE!
    - If an answer is even 10% wrong, then it is ALL wrong!
  - Direct Repetition
    - As stated above, the GRE loves to reproduce lines and jargon directly from the text. Of course, whereas correct answers will contain some direct language, often times direct repetitions are used only to trigger a false hit.
    - Don't fall for answer choices that quote the text verbatim – they are most likely acting as sucker answers.

DAY 5
TABLE OF CONTENTS

- Cylinders
- Cones and pyramids
- Diagonals

GEOMETRY I

1. Overview

Plane Geometry. Plane geometry is the study of shapes and figures in 2 dimensions ("flat" figures).

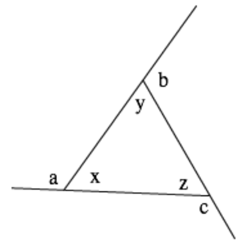
- Almost all geometry that you will encounter on the GRE is plane geometry.
- It is very important to keep in mind that except for the data analysis section all figures on the GRE are *not drawn to scale* unless noted otherwise.
- This means that all problems with figures are solvable by standard logical and mathematical methods and do not require estimations that rely on the diagrams. Furthermore, such estimations are often intentionally misleading (e.g., drawing an angle to look like a right angle, but then giving information that tells you it is acute or obtuse). On these questions it is often helpful to draw the two most extreme figures that are consistent with the information given to anchor the distribution of possibilities.
- You can assume that: (1) straight lines are indeed straight; (2) positions of points, angles, regions, etc. are in the order shown; (3) angle measurements are positive; (4) a circle is a circle, a triangle is a triangle, a quadrilateral is a quadrilateral, etc. (this is actually an extension of assumptions #1 & #2); (5) figures lie in a plane unless otherwise specified (but we have never seen anything not in a plane).

Solid Geometry. Solid geometry is the study of shapes and figures in 3 dimensions.

- It is not likely that you will see many problems that involve geometry of 3-dimensional figures.
- The 3-D geometry that you are responsible for is very simple.

2. Basic Geometry Vocabulary

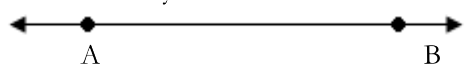
- Point: marks a location but technically occupies no space because it is 0-dimensional (it has no length, width, or depth). A point is signified with a dot.
- Line: a straight 1-dimensional geometrical object (it has length, but no width or depth). A line is infinitely long (although of course *line segments* are finite).
- Surface: a 2-dimensional figure that has length and width but no depth.
- Solid: a 3-dimensional figure that has length, width, and depth.
- Angle: the common endpoint formed by the intersection of 2 non-parallel lines.
  - Interior angle: an angle within a polygon (angles x, y, and z in the figure below).
  - Exterior angle: an angle outside of a polygon formed by one side of the polygon and a line extending from another side (angles a, b, and c in the figure below).
  - Vertex: the point at which the lines meet to make an angle (the "common endpoint").
- Congruent figures: two figures that are geometrically identical: that is, they have the same number of sides and angles. All sides in one figure are the *exact* same length as the corresponding sides in the other figure and all angles in one figure have the *exact* same measure as the corresponding angles in the other figure.
- Similar figures: have the same number of sides and angles. Like congruent figures, all of one figure's angles must be equal to the corresponding angles in the other figure. However, the sides need not be equal. They do need to be *proportional* though (i.e. two sides measuring 4 and 6 in one shape must have the same ratio in the other, such as 8 and 12).
- Polygon: a closed 2-dimensional figure that consists of 3 or more straight lines.
  - Triangle: a three-sided polygon
  - Quadrilateral: a four-sided polygon



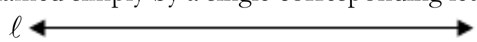
- Equiangular polygon: any polygon with congruent interior angles (e.g. a triangle with all angles equal to 60°).
- Equilateral polygon: any polygon whose sides are all congruent (e.g. a triangle with all sides equal to 3 inches).
    - An equilateral polygon must also be equiangular, but the reverse is not true (e.g., rectangles).

3. Lines

- Straight Line: the shortest distance between two points. It continues forever in both directions and consists of an infinite number of points.
    - A line can be notated in several ways:

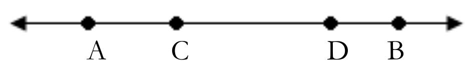


    - This line can be written as: "line AB" or $\overleftrightarrow{AB}$
    - A line may also be named simply by a single corresponding letter:



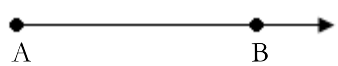
    - This line could be written as: Line ℓ or $\overleftrightarrow{\ell}$

- Line Segment: a piece of a line. It has two endpoints, which are used to denote the segment. The endpoints are represented as points on the line, and have letters attached to them for reference.

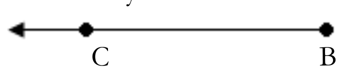


    - Line segment CD (and others like it) is written WITHOUT arrows: CD, or $\overline{CD}$

- Ray: a ray has only one endpoint and continues forever in the opposing direction. It is denoted first by the letter marking the endpoint and second by any other points in the infinitely extending direction.



    - This ray could be written as: Ray AB or $\overrightarrow{AB}$



    - This ray could be written as: Ray BC or $\overleftarrow{CB}$

- Midpoint: the point that divides a line into 2 equal lengths. Note that only a line *segment* can have a midpoint because a line is infinite.

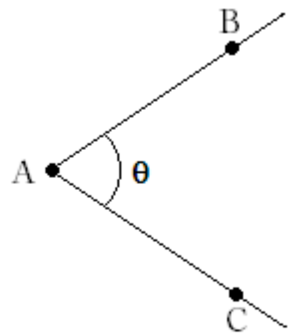
- Segment bisector: a line whose intersection with a line segment divides that segment into 2 equal lengths. A segment bisector is similar to a midpoint, except that it is a *line*, not a point. Segment bisectors need not be perpendicular to the segment they bisect (and since the figures are not drawn to scale, never assume the bisector is perpendicular unless it is explicitly stated). However, bisectors can never be parallel to the segment they intersect.

- Parallel lines: lines that never intersect. They can be represented with the symbol "||" such that if you have two lines AB and XY and the problem says "AB||XY," it means these lines are parallel (and as a result, they never intersect). This is typically important for figuring out angles.

- Perpendicular lines: perpendicular lines intersect at a 90° angle. They can be represented with the symbol "⊥" such that if you have two lines AB and XY and the problem says "AB⊥XY," it means these lines are perpendicular and thus all angles at the intersection are 90°.

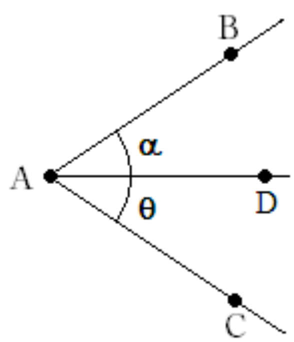
4. Angles

- Angle: the common endpoint formed by the intersection of 2 non-parallel lines.
  - Vertex: the point at which the lines meet to make an angle (the "common endpoint").
    - Angles are measured in degrees (a measurement which indicates the size of the angle between the two sides). The below diagram provides a brief refresher on common angles.
    - Angle notation:

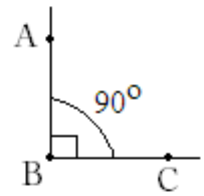


    - In the diagram above, point A is the vertex and is signified with the "∠" symbol.
    - The above diagram could thus be labeled: ∠BAC, ∠CAB, ∠A, or ∠θ.

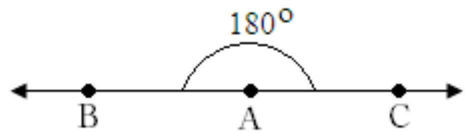
- Types of Angles
  - Adjacent angles: any angles that share a common side and a common vertex.
    - In the below diagram, ∠θ and ∠α are adjacent angles.



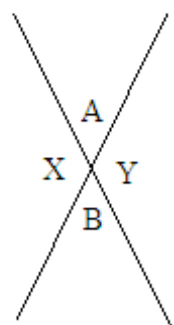
- Right angles: angles that have a measure of 90° and are formed by the intersection of two perpendicular lines. The symbol ⊾ will appear in the interior angle of a right angle. If you see this symbol it means the angle measures 90°.



  - In the above diagram, ∠ABC is a right angle.
- Acute angle: an angle whose measure is less than 90°.
- Obtuse angle: an angle whose measure is greater than 90° but less than 180°.
- Straight angle: an angle whose measure is 180°. The angle along any line is 180°, and all angles along a line must add up to 180°.

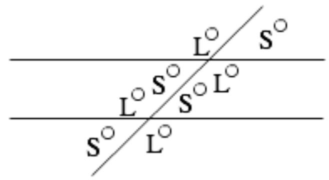


- Complementary angles: two angles whose sum is 90°.
- Supplementary angles: two angles whose sum is 180°.
- Vertical Angles: the angles directly across from each other when two lines intercept. Vertical angles are always equal!



  - In the above diagram, angles A and B are vertical (and equal) and angles X and Y are vertical (and equal).
  - In addition, Angles A & X; Angles X & B; Angles B & Y; and Angles Y & A are supplementary (they add to 180° because they sit on a line).

- The Parallel Line Diagram: when two lines are parallel and are intersected by a third line you only need to know 1 angle to know all 8. You can bet the GRE will test this.
  - When this happens, only two types of angles are formed. We'll call them:
    - Large Angles (L° in the diagram below)
    - Small Angles (S° in the diagram below)
      - All large angles are equal, and all small angles are equal!
      - The sum of any large angle and small angle equals 180° (supplementary).
      - Often when trying to figure out angles when working with parallel lines, writing this diagram out on your scratch paper can help you see everything more clearly.



- When two lines intersect, four angles are formed with a sum of 360°.

- Interior angle relationships
  - The three interior angles of a triangle add up to 180°.
  - The four interior angles of a quadrilateral add up to 360°.
  - The five interior angles of a pentagon add up to 540°.
  - For polygons with even more sides, add 180° for each additional side. Expressed as an equation, the angles in any polygon with *n* sides sum to (180(n – 2))°.

- Exterior angles always add to 360°, *no matter how many sides* the polygon has.

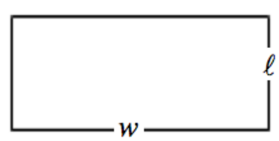
5. Perimeter, Area, and Quadrilaterals

- Perimeter: the perimeter of any non-circular figure is the sum of the lengths of all its sides, and is thus the total distance of the lengths of all sides.
  - Perimeters have the units of lengths (e.g. inches, feet, meters, etc.).

- Area: the area of a figure is the total amount of space covered by the surface bounded by all its sides. The area is always found by multiplying two sides together in one way or another.
  - Areas have the units of lengths squared (e.g. meters^2, inches^2, etc.).

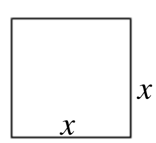
- Quadrilaterals: a quadrilateral is any polygon with four sides.
  - Rectangles: a rectangle is an equiangular quadrilateral, so all angles are equal to 90°.
    - If you are told something is a rectangle, you know all angles are 90°, and the sides of the lengths are equal as are the sides of the widths.
    - The area of a rectangle can be found by multiplying length by width, and the perimeter can be found by adding all sides or adding 2(length) + 2(width):

$$Area = \ell \times w$$
$$Perimeter = 2\ell + 2w$$



  - Squares: a square is an equilateral rectangle.
    - If you are told it is a square you know all angles are 90° and all sides are equal.
    - Squares can be bisected with a diagonal line to form two 90:45:45 triangles
    - The area of a square can be found by squaring the length of a side (this is why raising something to the power of 2 is called "squaring" it) and the perimeter can be found by multiplying a side by 4:

$$Area = x^2$$
$$Perimeter = 4x$$



- All squares are rectangles, but not all rectangles are squares; all rectangles and squares are quadrilaterals, but not all quadrilaterals are rectangles or squares.

AREA AND PERIMETER FORMULAS YOU MUST KNOW

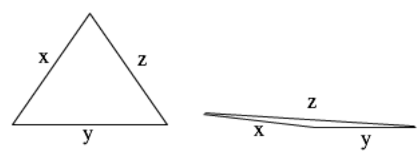
Shape	Area	Perimeter/Circumference
Rectangle (ℓ = length; w = width)	$A = \ell \times w$	$P = 2\ell + 2w$
Square (x = length of all sides)	$A = x^2$	$P = 4x$
Triangle (b = base; h = height)	$A = \frac{1}{2} b \times h$	No simple formula. Add all 3 sides.
Circle (r = radius; d = diameter)	$A = \pi \times r^2$	$C = 2\pi \times r = \pi \times d$

The area and perimeter formulas you need to know for the test are summarized here for convenience, but are explained in each of the figures' respective sections.

6. Triangles

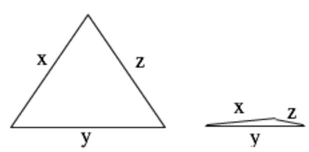
- Triangles: a triangle is a polygon with three sides.
  - There are a number of special types of triangles that all have unique properties (some examples: equilateral, isosceles, and right triangles). It is important to keep these all clear in your mind because what's true of one type of triangle may not be true of another.
  - Interior angles: no matter what a triangle looks like, its interior angles always add up to equal 180^o.
  - Triangle Angle-Side Relations: in any triangle, the longest side will be opposite the largest interior angle. Likewise, the shortest side will be opposite the smallest interior angle. Also, equal length sides will always be opposite equal degree angles.
  - The Third Side Rule: there are two parts to the third side rule, which results from the basic understanding that a triangle can never have an angle greater than 180° or less than 0° (because if it did, it would be a line rather than a triangle!).
    - Because there can never be an angle greater than 180° the sum of two sides of a triangle will always be greater than the length of the third side. If we knew x and y in the triangle below, we could see from the diagram on the right that z could not be greater than their sum.

$$z < x + y$$



    - Because there can never be an angle less than 0° the difference of two sides of a triangle will always be less than the length of the third side. If we knew x and y in the triangle below, we can see from the diagram on the right that z could never be less than the absolute value of their difference (it's absolute value because z can't be negative).

$$z > |y - x|$$



    - Notice that both parts of the third side rule use *greater than* and *less than* rather than *greater than or equal to* and *less than or equal to* because a triangle cannot have an angle of 180° or of 0°.

Example: A triangle has sides 4, 7, and x. Which of the following could be the triangle's perimeter?

A) 11
B) 14
C) 18
D) 22
E) 25

Because the perimeter is the sum of the sides or 7 + 4 + x, we must first figure out what x is.
The value of x is constrained by the third side rule:

x > |4 – 7| (here we see why we must have the absolute value of the difference)
x < 4 + 7   Thus:
3 < x < 11

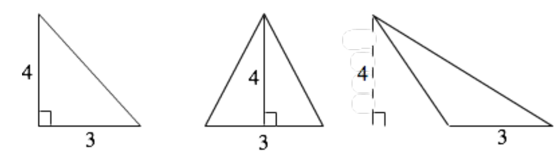
Now we can solve for the constraints of the perimeter by plugging in these boundary conditions.

7 + 4 + 3 = 14 and 7 + 4 + 11 = 22
Since x cannot be equal to 3 or 11, we know: 14 < perimeter < 22, leaving only C

Answer: C) 18

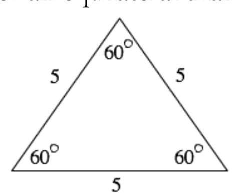
- <u>Area of a Triangle</u>: the area of any triangle is equal to the height multiplied by the base divided by 2 (also known as ½bh).
  - This formula is derived from the fact that any triangle occupies half of the space of a rectangle with corresponding dimensions.
  - It is important to realize that the height is the distance of a perpendicular line going from the base to the top of the triangle, even if the triangle is obtuse so that the peak of the triangle is not actually over the base.

$$Area = \frac{1}{2} b \times h$$

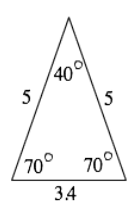


Area = 6   Area = 6   Area = 6

- <u>Equilateral Triangles</u>: an equilateral triangle has three sides of equal length.
  - The three interior angles of an equilateral triangle are also equal and are always 60°.

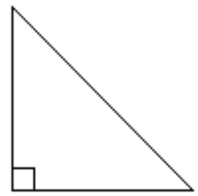


- <u>Isosceles Triangles</u>: an isosceles triangle is a triangle in which two of the three sides are equal.
  - Because of this, the two angles opposite the equal sides must also be equal.
  - If you know the degree measure of any one angle in an isosceles triangle then you can calculate the other two.



7. Right Triangles

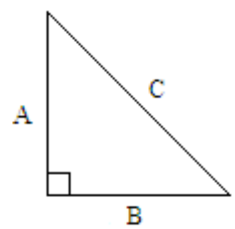
- <u>Right Triangles</u>: a right triangle is a triangle with one angle with a measure of 90°. The other two angles must also sum to 90°.



- Hypotenuse: the hypotenuse is the side opposite the 90° angle. It is always the longest side (because it is always across from the largest angle).
- Legs: the legs are the other two sides, opposite from the smaller angles.
- Right triangles are so special that they get their own section here. This is because they have a number of unique properties that *only apply to right triangles*. Thus you cannot use any of the relationships below unless the triangle has a 90° angle!
- On the GRE a right triangle will always have the ∟ 90° marker,

- The Pythagorean Theorem: the square of the length of the hypotenuse (the longest side, opposite the 90° angle) is equal to the sum of the squares of the lengths of the other two sides.

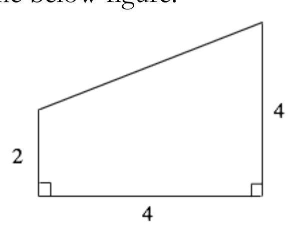
$$A^2 + B^2 = C^2$$
$$C = \sqrt{A^2 + B^2}$$
$$A = \sqrt{C^2 - B^2}$$



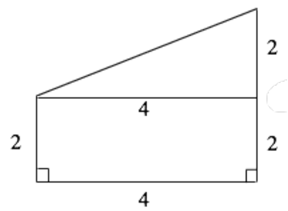
BEWARE: Now is no time to forget your rules for roots. The GRE might set you up with tempting numbers to make it seem like you should do an operation that is not allowed. Remember: $\sqrt{A^2 + B^2} \neq \sqrt{A^2} + \sqrt{B^2}$. (Think about the logic of this rule: there would be no point to splitting addition and subtraction under a radical in such a fashion because $\sqrt{A^2} = A$!)

- The typical use of the Pythagorean theorem is to solve for the length of some unknown side given the other two.
- As stated earlier, the Pythagorean theorem applies only to right triangles, which means you can also use this relationship to test whether a triangle with unknown angles is a right triangle: if you know the lengths of all 3 sides of a triangle and the square of the hypotenuse is equal to the sum of the square of the legs, it must be a right triangle (this is the kind of application the GRE loves to test!).
- Problems on the GRE often involve pulling out right triangles or creating your own within seemingly more difficult shapes as in the below example. Always be on the lookout for right triangles!

Example: What is the perimeter of the below figure?



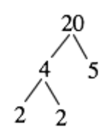
Since the perimeter is the sum of all the sides, we must find the length of the missing side first. You have no formula for finding that side in this weird figure. This means you must split this strange shape into shapes you are familiar with. Hopefully, you see that it can be broken into a *right triangle* and a *rectangle*. Don't forget to put in all relevant measurements.



Now that we have a right triangle, we can use Pythagorean theorem to solve for the missing side.

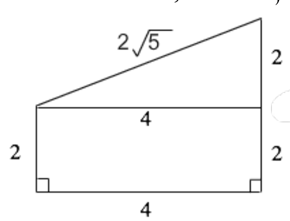
$$C = \sqrt{4^2 + 2^2} = \sqrt{20}$$

Great: now we have that missing side! However, you will never see $\sqrt{20}$ in an answer because it is not simplified. Remember: to simplify, we do prime factorization and pull all factor pairs out of the radical.



Since we have two 2s, we pull them out front to get $2\sqrt{5}$.

Now we're in business. Our figure is as shown below, and we just add all sides to get the perimeter.

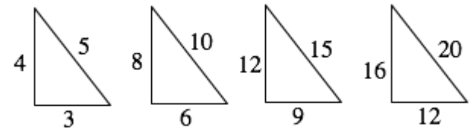


Perimeter = $2 + 2 + 4 + 2 + 2\sqrt{5} = 10 + 2\sqrt{5}$

Answer: $10 + 2\sqrt{5}$

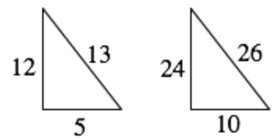
- Special Right Triangles: these are right triangles with known proportional relationships that you will need to memorize for the test.
  - Tao of the GRE™ applied: if you ever see shapes with the common values of these triples, you will likely need your special triangle knowledge to solve the problem. You should have both the basic proportions and few common instances of values used (see table below) memorized.
  - Several right triangles are called Pythagorean triples because the sides are all integers.
    - It is important to realize that there are elementary triples such as 3-4-5 and that all multiples of these are also special triangles (e.g. 6-8-10; 9-12-15)
    - Because all multiples are also special triangles, you should learn them as ratios for proportions. In the following formulas x can have any integer value as long as it is the same for all three sides. The common variants you need to memorize are in the diagrams on the right (with corresponding x values):

- 3x : 4x : 5x



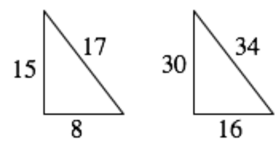
x =1 x = 2 x =3 x = 4

- 5x : 12x : 13x



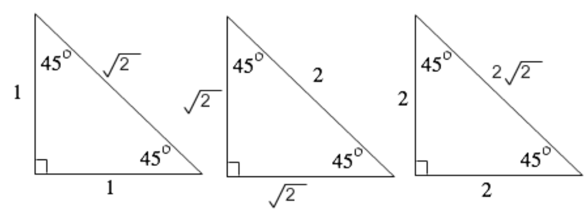
x = 1 x = 2

- 8x : 15x : 17x



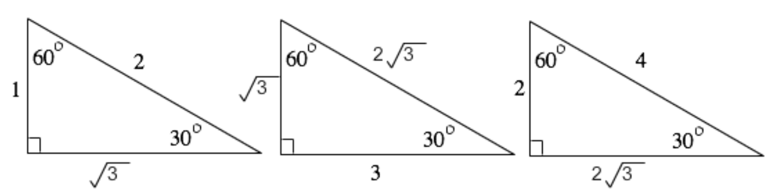
x = 1 x = 2

- In addition to Pythagorean triples, there are two other special right triangles you will likely see on the exam. For these you must know both sides and angles, and importantly, which sides are opposite which angles.
  - <u>45°– 45°– 90° right triangle</u> (also called an Isosceles Right Triangle: the shortest sides (x) is across from the smallest angles (45°), and longest side ($\sqrt{2}$) is across from 90° angle.
  - $x : x : x\sqrt{2}$



x = 1 x = $\sqrt{2}$ x = 2

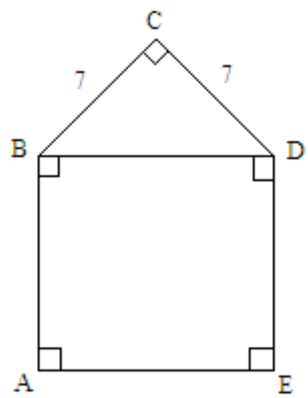
  - <u>30° – 60° – 90° right triangle</u>: the longest side (2x) is across from the 90° angle and the shortest (x) is across from the 30° angle.
  - $x : x\sqrt{3} : 2x$



x = 1 x = $\sqrt{3}$ x = 2

- TAKE NOTE: You may notice that many of the same numbers pop up in different places in different triangles. Be careful that you know exactly which sides have which numbers. For example with a 45°– 45°– 90° triangle, you will notice that $\sqrt{2}$ appears both in the triangle where x = 1 and the one in which x = $\sqrt{2}$; however, in the first this number applies to the hypotenuse, while in the second it applies to the legs!
    - Sometimes the GRE will try to trick you into thinking you have a special triangle by putting the numbers on the wrong parts. For example, if a right triangle has one leg with a length of 4 and another with a length of 5, it is not a special triangle (for which 5 would have to be the length of the hypotenuse). Its hypotenuse would not equal 3, but rather $\sqrt{41}$.

Example: In the figure below, what is the area of square ABDE?



First we need to find the length of the square's sides. Because BCD is a right triangle and both its legs are the same length, it must be a 45°– 45°– 90° triangle.

Thus the length of a side must be $7\sqrt{2}$ due to the $x : x : x\sqrt{2}$ relationship.

The area of a square is just x^2.

$$\left(7\sqrt{2}\right)^2 = 7^2 \times \left(\sqrt{2}\right)^2 = 49 \times 2 = 98$$

Answer: 98

Your turn:

Example: Triangle XYZ is equilateral. If the perimeter is 12, what is the area?
(*Hint: you don't know any helpful relationships for equilateral triangles, but you do for 30° – 60° – 90° triangles*)

Answer: $18\sqrt{3}$

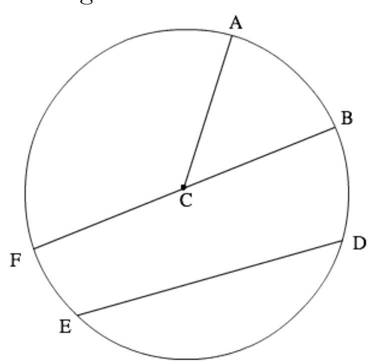
COMMON SPECIAL TRIANGLE VALUES YOU NEED TO MEMORIZE

Formulas	Angles	x = 1	x = 2	x = 3	x = 4	x = 5
$3x:4x:5x$		3-4-5	6-8-10	9-12-15	12-16-20	15-20-25
$5x:12x:13x$		5-12-13	10-24-26			
$8x:15x:17x$		8-15-17	16-30-34			
				x = $\sqrt{2}$	x = $\sqrt{3}$	
$x:x:x\sqrt{2}$	45°-45°-90°	1-1-$\sqrt{2}$	2-2-$2\sqrt{2}$	$\sqrt{2}$-$\sqrt{2}$-2		
$x:x\sqrt{3}:2x$	30°-60°-90°	1-$\sqrt{3}$-2	2-$2\sqrt{2}$-4		$\sqrt{3}$-3-$2\sqrt{3}$	

GEOMETRY II

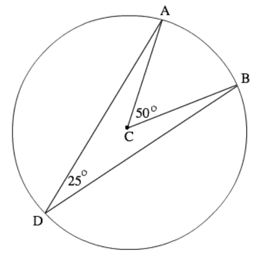
8. Circles

- Basic circle vocabulary: see diagram below definitions for examples of each. Each definition has a reference to the diagram beneath.
  - Circle: a circle consists of a *curved line* in which *every point on the line is equidistant from the center.*
  - Center: the point that all points on the curved line of the circle are equidistant from. Thus any point on the circle is the exact same distance from the center as any other point.
    - *Point C in diagram.*
  - Arc: any part of the curved line that the circle consists of.
    - *There are many arcs in the diagram: AB, AD, AE, AF, BD, DE, EF, etc.*
  - Radius: any line that extends from the center of a circle to the edge of a circle.
    - *There are 3 in the diagram: AC, BC, FC.*
  - Sector: any part of the surface area of a circle bordered by two radii and an arc (like a slice of pie).
    - *There are a number in the diagram: ABC, FCA, etc.*
  - Chord: any line that connects two points on the circle. The further that a chord is from the center of the circle, the shorter it is. Thus, the diameter is the longest chord of a circle.
    - *Both FB and ED are chords (but CA is not because it stops at the center). FB is longer than ED.*
  - Diameter: any line that extends from one edge of a circle to the other edge and *passes through the center*. The diameter of a circle is twice as long as its radius and is the longest chord in the circle.
    - *FB is the only diameter in the diagram.*



- Angles and circles
  - Central angle: any angle whose vertex sits on the center point of a circle.
    - *This is ∠ACB in the below diagram.*
    - The arc carved out by a central angle is equal to the measure of that angle.
    - The measure of an arc can be used to find its length if you know the circumference of the circle: multiply the total circumference by the ratio of the arc's measure to 360°.
  - Inscribed angle: an angle is whose vertex sits on the curved line (the edge) of the circle.
    - *This is ∠ADB in the diagram below.*
    - The arc carved out by an inscribed angle is equal to twice the measure of the angle.

The arc AB has a measure of 50°, and thus the length of the arc would be equal to (50/360)(Circumference)



- Pi (π): the ratio between the circumference of a circle and its diameter.
  - π = 3.14 ≈ 3 1/7
- Circumference: the distance around the outside of the circle. It is analogous to the perimeter of a polygon. Polygons have perimeter; circles have circumference.
  - Circumference is found by multiplying π by the diameter (or twice the radius).

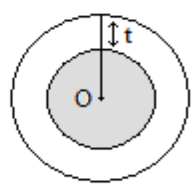
$$Circumference = \pi \times d = \pi \times 2r$$

- Area: the area of a circle is the space within a circle.
  - A circle's area is found by multiplying π by a circle's radius squared.

$$Area = \pi \times r^2 = \pi \times \frac{d^2}{4}$$

- Typical GRE circle questions
  - The GREs will never simply test you on your knowledge of one or two of the above properties. Rather, you will be asked questions that require you to jump between and relate ALL of the above properties continuously.
  - REMEMBER: If you know even ONE of the above variables (radius, diameter, circumference, area), you can find all three of the others.

Example: In the wheel below, with center O, the area of the entire wheel is 169π. If the area of the shaded hubcap is 144π, then what is the value of t?



To solve this we must use our total knowledge of circles and their components. Let's start with what information we know: The entire wheel had an area of 169π. We can use the area to find the radius (notice that 169 and 144 are both perfect squares; this is a hint that you will take a square root somewhere in the problem, and that is exactly what this step requires).

$$Area = \pi \times r^2 = 169\pi$$

$$r^2 = \frac{169\pi}{\pi} = 169$$

$$r = \sqrt{169} = 13$$

So we know the radius of the large circle is 13. What else do we know? The area of the shaded hubcap is 144π. Let's put that information through the same process to get the radius of the smaller circle:

$$r^2 = \frac{144\pi}{\pi} = 144$$

$$r = \sqrt{144} = 12$$

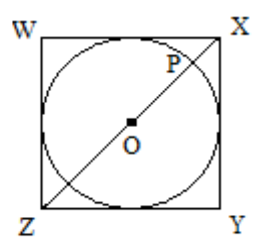
Now that we have the radii of both circles, we just need to subtract to get t.

t = 13 – 12 = 1

Answer: t = 1

Your turn:

Example: In the figure below, a circle with center O is inscribed in square WXYZ. If the circle has radius 3, then what is the length of $\overline{PZ}$?



Answer: $\overline{PZ} = 3 + 3\sqrt{2}$

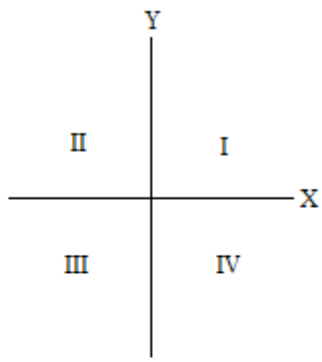
9. Coordinate Geometry (aka Analytical Geometry)

- Cartesian Coordinate Plane: the Cartesian coordinate plane is just a fancy term for the standard graph that you are undoubtedly familiar with. It is composed of 2 axes:
  - The horizontal line is called the X-Axis
  - The vertical line is called the Y-Axis
  - Origin: the origin is the point where the two axes intersect, and has a value of 0 for both the X and Y axes, and would thus be signified as (0, 0).

- Ordered pairs: any point within the coordinate system can be expressed by first giving the X value and then the Y value, like this: (X, Y). These are called ordered pairs, and they must always be in this order, with the X value first and the Y value second.

- Both axes are number lines, with the origin being 0 for both
- The X-axis values are positive to the right of the origin and negative to the left
- The Y-axis values are positive above the origin and negative below

- Quadrants: the two axes divide the coordinate system into 4 quadrants (labeled I, II, III, and IV):



- Quadrant I always has a positive X and positive Y value (+, +)
- Quadrant II always has a negative X and Positive Y value (–, +)
- Quadrant III always has a negative X and negative Y value (–, –)
- Quadrant IV always has a positive X and negative Y value (+, –)

- Slope: the slope of a line is the ratio of the line's vertical rise to the line's horizontal run, and is represented by the letter *m*. This is often captured by the phrase "rise over run."
  - The slope represents the relationship between the two variables your line represents (x and y). It shows how these variables change together. For example, if the slope of your line is 2 that would mean that when the value of x goes up 1, the value of y goes up 2.
  - The slope of a line can be found by picking any two points, say (x_1, y_1) and (x_2, y_2), and then taking the difference in their Y values and dividing it by the difference in their X values.

$$Slope = \frac{Rise}{Run}$$

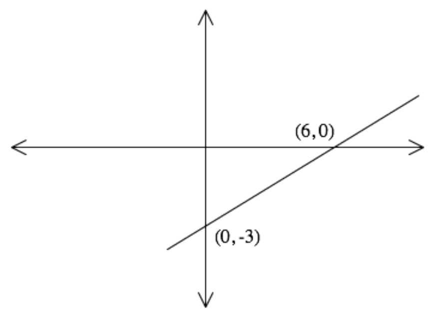
$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

  - In the above equation it doesn't matter which y-coordinate is first and which is second in the subtraction. The same goes for the x-coordinates. However, it does matter that you are consistent. Don't do this, because it will give you the wrong sign (positive slope when it should be negative or vice versa): $\frac{y_2 - y_1}{x_1 - x_2}$
  - Lines that run from bottom left to upper right have a positive slope (/).
  - Lines that run from bottom right to upper left have a negative slope (\ ).
  - Horizontal lines have a slope of 0.
  - Vertical lines have an undefined slope (because there is no horizontal change, so the equation has a 0 in the denominator).

- Parallel and Perpendicular Slopes
  - Parallel lines have the same slope. Lines can only be parallel if they have the same slope.
  - Perpendicular lines have slopes that are negative reciprocals of each other (e.g. 4 and – 1/4). That is, if line A is perpendicular to line B, its slope must have both the opposite sign as the slope of line B, and it must be the reciprocal of the slope of line B. Another way of saying this is that the product of the slopes of perpendicular lines must be –1.

- Intercepts: an intercept is the point where a line hits either axis. Every line that is not horizontal or vertical has an x-intercept and a y-intercept.
  - At the x-intercept, the value of the y-coordinate must be 0. This is because the x-axis sits at the value of 0 on the coordinate plane. Thus the ordered pair at the x-intercept will always be (x, 0), where x is the value of the x-intercept. *The x-intercept is (6, 0) in the below graph.*
  - Likewise, at the y-intercept, the x-coordinate will always be 0 for the same reason. The ordered pair at the y-intercept will always be (0, y), where y is the value of the y-intercept. *The y-intercept is (0, -3) in the below graph.*

$$m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{0-(-3)}{6-0} = \frac{1}{2}$$



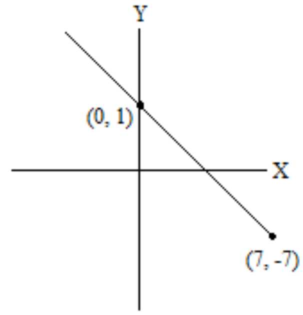
- Slope Intercept Equation: this is the form that you will want to get all linear equations into if you are dealing with graphing them. Any linear equation can be turned into a slope-intercept equation, which takes the following form:

$$y = mx + b$$

With numbers this will look like: $y = 4x + 3$

- y = the y value of a specified point
- m = the slope of the line
- x = the x value of the same specified point
- b = the value of the y-intercept
  - You should note that the value of b falls directly out of the definition of a y-intercept. The y-intercept is the point in the line where x = 0, and in the above equation when x = 0, y = b.

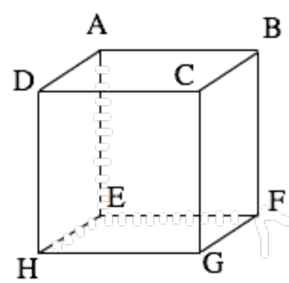
Example: What is the equation of the line shown below?



Answer: $y = -\frac{8}{7}x + 1$

10. Three Dimensional Figures

- Basic 3-D Figure Vocabulary: all terms are illustrated on the figure at the bottom of the section.
  - Solid: the general term for a 3-D figure (e.g. a 3-D rectangle is called a rectangular solid).
  - Face: a 2-dimensional surface that bounds a 3-D figure. These form the boundaries of solids: just as 1-D *lines* form the boundaries of 2-D figures, 2-D *surfaces* form the boundaries of 3-D figures.
    - *The figure below has 6 faces including ABCD, EFGH, CBFG, etc.*
  - Edge: the line segment that results from the meeting of 2 faces.
    - *The figure below has 12 edges including AB, BC, CG, EF, HD, etc.*
  - Vertex: any point where 2 edges meet or cross.
    - *The below figure has 8 vertices; all points with the letter labels are vertices.*
  - Base: the face from which the *height* is measured (see formula below).
    - *The base really depends on what is used for height, but the obvious one below would be EFGH (with height being the length of the vertical edges such as CG or BF).*
  - Lateral surface: any face that is not the base.
    - *If EFGH is the base, then all other faces are lateral surfaces including AEHD, ABCD, CBFG, etc.*



- Volume: volume refers to the capacity a 3-D figure can hold (analogous to area for a 2-D figure). This can also be thought of as the amount of area that a figure can contain (see below illustration). All volumes have the units of lengths-cubed (e.g. meters^3, inches^3, etc.)
  - All formulas for volume that you will need to know take the general form of multiplying the area of a 2-dimensional slice by the number of slices in the figure (which would be the height).
  - The general formula for volume for all figures is: $Volume = Area \times Height$

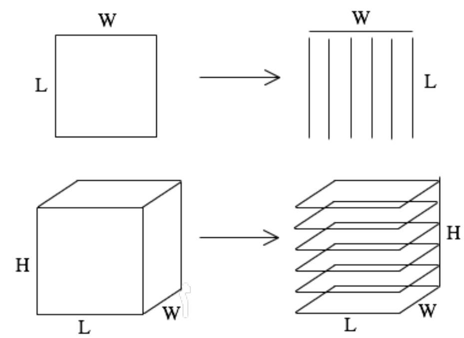
- A conceptual analysis of area and volume
  - Area is a 2-D quantity, and all formulas for area involve multiplying 2 lengths. This can be

thought of as taking some 1-D line segment that has a given length, L, and then multiplying by the number of those in the shape, which is represented by the width, W. This decomposition is illustrated in the top row below. (This is not literally true because a line has no width, but it is a helpful way to understand the calculation.)

- Volume is a 3-D quantity, and all formulas for volume involve multiplying a 2-D area by a 1-D length. This can be thought of as taking the 2-D area (A = L x W) of the base, and multiplying it by the number of these shapes that are stacked to make the solid, which is represented by multiplying by the height, H. This illustration is in the bottom row. (Again, not literally true.)

If a square has a side length L, in some sense the area is a measure of all those lengths added together (A = L + L + L +…). If there are W lengths then this is A = L x W!

If a solid has a base with area A, the volume is similar to a measure of all those areas added together (V = A + A + A +…). If there are H bases this is V = A x H

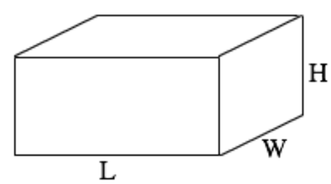


- Surface Area: the surface area of a 3-D figure is the sum of all the 2-D surfaces that form the 3-D solid. Because surface areas are areas, they have the units of lengths-squared (e.g. meters^2, etc.)

- Rectangular Solids: are just what they sound like, a 3-D figure with a rectangle base and with all angles equal to 90°.
  - Volume: the volume of a rectangular solid is the length times the width times the height.
    - Volume can also be found with area x height because (A = L x W).
  - Surface Area: the sum of the areas of all the faces. There are 3 unique faces, and there are 2 faces of each kind. You must find the area of all three unique faces (A_1 = L x W; A_2 = W x H; A_3 = L x H), and then multiply all by 2 and sum.

$$Volume = L \times W \times H = A \times H$$

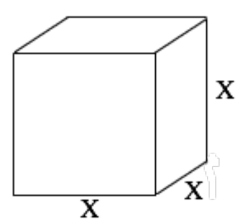
$$Surface\ Area = 2LW + 2WH + 2LH$$



- Cubes: cubes are a special type of rectangular solid in which all sides are equal length.
  - Volume: the volume of a cube is the length of a side cubed (this is why taking something to the 3rd power is called "cubing" it). Notice that this means the volume of a cube with integer side lengths must always be a perfect cube
  - Surface Area: the sum of the areas of all the faces. Because all sides are the same length you just need to multiply the area of one face by 6.

$$Area = x^3$$

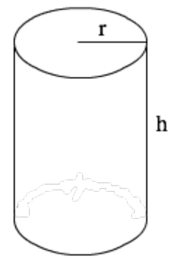
$$Surface\ Area = 6x^2$$



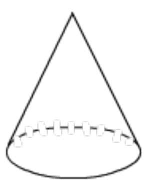
- <u>Cylinders</u>: the faces of a cylinder consist of two circles (top and bottom) and a rectangle that is essentially wrapped around the circles.
  - <u>Volume</u>: the volume of a cylinder is the area of one of the circles (πr^2) times the height.
  - <u>Surface Area</u>: the surface area is the sum of the areas of the two circles at either end and that of the rectangle between them. The area of the rectangle is length x width, where the length is equal to the height of the cylinder (h) and the width is equal to the circumference of the circles ($2\pi r$).
    - If you are having trouble seeing the rectangle imagine taking a paper towel roll (which is a cylinder without the ends), and cutting it lengthwise to open it up. If you did this you would get a rectangle with a length equal to the height of the cylinder, and a width equal to the circumference of one of the circles.

$$Volume = (\pi \times r^2) \times h = A_{circle} \times h$$

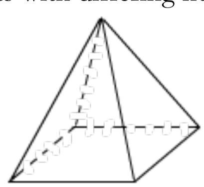
$$Surface\ Area = 2(\pi \times r^2) + (2\pi \times r) \times h$$
$$Surface\ Area = 2(Area_{Circle}) + (Circumference \times h)$$



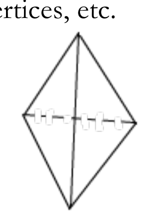
- <u>Cones and Pyramids</u>: you might also be asked conceptual questions about cones and pyramids, but you do not need to know how to calculate volume or surface area. Familiarity with the figures shown below is sufficient.
  - Cones have a circular base.
  - Pyramids have a polygon base. It is important to note that different polygon bases create qualitatively different pyramids with differing numbers of edges, vertices, etc.



Cone



Pyramid with quadrilateral base



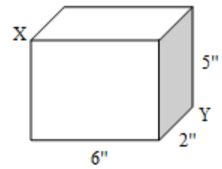
Pyramid with triangle base

- <u>Diagonals</u>
  - A question you may possibly get will refer to the length of a diagonal inside a 3-D rectangular box (such a question will often be worded "find the longest distance between two corners").
    - The formula to find this distance is an addendum to the Pythagorean theorem:

$$A^2 + B^2 + C^2 = D^2$$

      - A = Length
      - B = Width
      - C = Height
      - D = Length of Diagonal
    - REMEMBER: just like the Pythagorean theorem, this equation gives you the length of the diagonal squared! You must take the square root to obtain the final answer.

Example: What is the distance between points X and Y in the below diagram?



Answer: $5\sqrt{3}$

GENERAL ROOTS AND PREFIXES

Root or Prefix	Meaning	Examples
a, an	not, without	atheist, anarchy, anonymous apathy, aphasia, anemia, atypical, anesthesia
ab	away from	absent, abduction, aberrant, abstemious, abnormal, abstract, absorb
acro	high, tip, top	acrobat, acrophobia, acronym, acromegaly
ad	to, toward	admit, addition, advertisement, adherent, admonish, address, adhesive, adept, adjust
alt	high	altitude, altimiter, alto, contralto, altocumulus
ambul	to walk	ambulatory, amble, ambulance, somnambulist, perambulate, preamble
ante	before	anteroom, antebellum, antedate antecedent, antediluvian
anti, ant	against, opposite	antisocial, antiseptic, antithesis, antibody, antichrist, antinomies, antifreeze, antipathy, antigen, antibiotic, antidote, antifungal, antidepressant
audi	to hear	audience, auditory, audible, auditorium, audiovisual, audition, audiobook
auto	self	automobile, automatic, autograph, autonomous, autoimmune, autopilot, autobiography
be	thoroughly	bedecked, besmirch, besprinkled, begrudge, begrime, belie, bemoan
bell	war	belligerent, antebellum, bellicose, rebel
bene	good, well	benefactor, beneficial, benevolent, benediction, beneficiary, benefit
bi	two	bicycle, bifocals, biceps, billion, binary, bivalve, bimonthly, bigamy, bimetal, biathlete, bicarbonate
cede, ceed, cess	to go, to yield	succeed, proceed, precede, recede, secession, exceed, succession, excess
chron	time	chronology, chronic, chronicle, chronometer, anachronism
cide, cis	to kill, to cut	fratricide, suicide, incision, excision, circumcision, precise, concise, precision
circum	around	circumnavigate, circumflex, circumstance, circumcision, circumference, circumorbital, circumlocution, circumvent, circumscribe, circulatory
clud, clus claus	to close	include, exclude, clause, claustrophobia, enclose, exclusive,

		reclusive, conclude, preclude
con, com	with, together	convene, compress, contemporary, converge, compact, confluence, concatenate, conjoin, combine, convert, compatible, consequence
contra, counter	against, opposite	contradict, counteract, contravene, contrary, counterspy, contrapuntal, contraband
cred	to believe	credo, credible, credence, credit, credential, credulity, incredulous, creed, incredible
commun	to share	commune, community, communism, communicable, communication, commonality, incommunicado
cycl	circle, wheel	bicycle, cyclical, cycle, encliclical, motorcycle, tricycle, cyclone
de	from, down, away	detach, deploy, derange, deodorize, devoid, deflate, degenerate, deice, descend, derail, depress, depart, decompose, destruction
dei, div	God, god	divinity, divine, deity, divination, deify
demo	people	democracy, demagogue, epidemic, demographic
dia	through, across, between	diameter, diagonal, dialogue, dialect, dialectic, diagnosis, diachronic, diagram, diaphragm
dict	speak	predict, verdict, malediction, dictionary, dictate, dictum, diction, indict, contradict
dis, dys, dif	away, not, negative	dismiss, differ, disallow, disperse, dissuade, disconnect, dysfunction, disproportion, disrespect, distemper, distaste, disarray, dyslexia
duc, duct	to lead, pull	produce, abduct, product, transducer, viaduct, aqueduct, induct, deduct, reduce, induce
dyn, dyna	power	dynamic, dynamometer, heterodyne, dynamite, dynamo, dynasty
ecto	outside, external	ectomorph, ectoderm, ectoplasm, ectopic, ectothermal
endo	inside, withing	endotoxin, endoscope, endogenous
equi	equal	equidistant, equilateral, equilibrium, equinox, equitable, equation, equator
e, ex	out, away, from	emit, expulsion, exhale, exit, express, exclusive, enervate, exceed, explosion
exter, extra	outside of	external, extrinsic, exterior, extraordinary, extrabiblical, extracurricular, extrapolate, extraneous, exterminator, extract, extradite, extraterrestrial, extrasensory, extravagant, extreme
flu, flux	flow	effluence, influence, effluvium, fluctuate, confluence, reflux, influx
flect, flex	to bend	flexible, reflection, deflect, circumflex, inflection, reflex
graph, gram	to write	polygraph, grammar, biography, graphite, telegram, autograph, lithograph, historiography, graphic, electrocardiogram, monogram
hetero	other	heterodox, heterogeneous, heterosexual, heterodyne

homo	same	homogenized, homosexual, homonym, homophone
hyper	over, above	hyperactive, hypertensive, hyperbolic, hypersensitive, hyperventilate, hyperkinetic, hyperlink, hypertext, hypersonic, hypertrophy
hypo	below, less than	hypotension, hypodermic, hypoglycemia, hypoallergenic, hypothermia, hypothesis
in, im	not	inviolate, innocuous, intractable, innocent, impregnable, impossible
infra	beneath	infrared, infrastructure, infrasonic
inter, intro	between	international, intercept, intermission, interoffice, internal, intermittent, introvert, introduce
intra	within, into	intranet, intracranial, intravenous, intramural, intramuscular, intraocular
jac, ject	to throw	reject, eject, project, trajectory, interject, dejected, inject, ejaculate
mal	bad, badly	malformation, maladjusted, dismal, malady, malcontent, malfeasance, maleficent, malevolent, malice, malaria, malfunction, malignant
mega	great, million	megaphone, megalomaniac, megabyte, megalopolis
meso	middle	mesomorph, mesoamerica, mesosphere
meta	beyond, change	metaphor, metamorphosis, metabolism, metahistorical, metainformation, metacognitive
meter	measure	perimeter, micrometer, ammeter, multimeter, altimeter, geometry, kilometer
micro	small	microscope, microprocessor, microfiche, micrometer, micrograph
mis	bad, badly	misinform, misinterpret, mispronounce, misnomer, mistake, misogynist
mit, miss	to send	transmit, permit, missile, missionary, remit, admit, missive, mission
morph	shape	polymorphic, morpheme, amorphous, metamorphosis, morphology, morphing
multi	many	multitude, multipartite, multiply, multipurpose, multicolored, multimedia, multinational
neo	new	neologism, neonate, neoclassic, neophyte
non	not	nonferrous, nonabrasive, nondescript, nonfat, nonfiction, nonprofit, nonsense, nonentity
omni	all	omnipotent, omnivorous, omniscient, omnibus, omnirange, omnipresent
para	beside	paraprofessional, paramedic, paraphrase, parachute, paralegal, parallel, comparison
per	through, intensive	permit, perspire, perforate, persuade, perceive, perfect, permit,

		perform
peri	around	periscope, perimeter, perigee, periodontal
phon	sound	telephone, phonics, phonograph, phonetic, homophone, microphone
phot	light	photograph, photosynthesis, photon
poly	many	polytheist, polygon, polygamy, polymorphous
port	to carry	porter, portable, report, transportation, deport, import, export
re	back, again	report, realign, retract, revise, regain, reflect, rename, restate, recombine, recalculate, redo
retro	backwards	retrorocket, retrospect, retrogression, retroactive
sanct	holy	sanctify, sanctuary, sanction, sanctimonious, sacrosanct
scrib, script	to write	inscription, prescribe, proscribe, manuscript, conscript, scribble, scribe
sect, sec	cut	intersect, transect, dissect, secant, section
semi	half	semifinal, semiconscious, semiannual, semimonthly, semicircle
spect	to look	inspect, spectator, circumspect, retrospect, prospect, spectacle
sub	under, below	submerge, submarine, substandard, subnormal, subvert, subdivision, submersible, submit
super, supra	above	superior, suprarenal, superscript, supernatural, supercede, superficial, superhero, superimpose
syn	together	synthesis, synchronous, syndicate, synergy, snyopsis, syncretism
tele	distance, from afar	television, telephone, telegraph, telemetry, telepathy
theo, the	God	theology, theist, polytheist, pantheism, atheist
therm, thermo	heat	thermal, thermometer, thermocouple, thermodynamic, thermoelectric
tract	to drag, draw	attract, tractor, traction, extract, retract, protract, detract, subtract, contract, intractable
trans	across	transoceanic, transmit, transport, transducer
un	not	uncooked, unharmed, unintended, unhappy
veh, vect	to carry	vector, vehicle, convection, vehement
vert, vers	to turn	convert, revert, advertise, versatile, vertigo, invert, reversion, extravert, introvert
vita	life	vital, vitality, vitamins, revitalize

A HANDFUL OF SUFFIXES

Suffix	Meaning	Examples
able, ible	able to be, capable of being	pourable, drinkable, readable, washable, curable, visible, flexible, collectible
ance, ancy	state of, process of	performance, reliance, defiance, radiance, acceptance, ascendancy, discrepancy, infancy
dom	condition, office, state	kingdom, freedom, wisdom, sheikdom, fiefdom, sheikdom
ee	one who receives	payee, mortgagee, employee, appointee, abductee, examinee, referee, refugee
er, or	one who does [the verb]	driver, hiker, reader, manager, polisher, speaker, counselor, author, creator, director, sculptor
ful	filled with	frightful, delightful, wonderful, cupful, wakeful, bashful, bountiful, beautiful, cheerful, colorful, dreadful, fateful
ify	to make into	purify, deify, simplify, clarify, petrify
ification	process of making into	purification, deification, simplifiction, clarification, petrification
ish	the nature of, resembling	Cornish, Irish, bookish, freakish, foolish, boorish, selfish, sluggish, priggish
ism	doctrine, system, characteristic quality	capitalism, heroism, optimism, skepticism, realism, patriotism, communism, idealism, conservatism
ist	one who performs; an adherent of an ism	tympanist, cellist, idealist, communist, realist, moralist, pharamacist, pragmatist
ize	to make into	rationalize, normalize, realize, capitalize
ization	the process of making into	rationalization, normalization, realization, capitalization
less	without	bottomless, effortless, friendless, noiseless, harmless